# Asset Pricing with a Financial Sector

Kai Li

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**Abstract**

This paper studies the quantitative asset pricing implications of financial intermediary which faces a leverage constraint. I use a recursive method to construct the global solution that accounts for occasionally binding constraint. Quantitatively, the model generates a high and countercyclical equity premium, a low and smooth risk-free interest rate, and a procyclical and persistent variation of price-dividend ratio, despite an independently and identically distributed consumption growth process and a moderate risk aversion of 10. As a distinct prediction from the model, when the intermediary is financially constrained, interest rate spread between interbank and household loans spikes. This pattern is consistent with the empirical evidence that high TED spread coincides with low stock price and high stock market volatility, which I confirm in the data.

Keywords: Financial Intermediary, Equity Premium, Return Predictability, TED spread, Global Method

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Department of Finance, School of Business and Management, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. Phone: (852)2358-8202; Email: kaili@ust.hk. I am deeply indebted to Ravi Bansal, Hengjie Ai, Tim Bollerslev, and Mariano Croce for their invaluable guidance and support. I would also like to thank Ian Dew-Becker, Michael Gallmeyer, Philipp Illeditsch (EFA discussant), Jia Li, Lukas Schmid, James Speckart, George Tauchen, Christopher Timmins, Harold Zhang (AsianFA discussant) for their helpful comments, and other seminar participants at Duke Macro Finance Reading Group and Duke Financial Econometrics Lunch Group, 2013 EFA meeting, 2013 AsianFA Conference, 2013 Asian meeting of Econometric Society. All remaining errors are my own. The most recent version of this paper can be found at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2219200

# 1 Introduction

This paper studies the quantitative asset pricing implications of financial intermediary[1]. I embed a financial intermediary sector with a leverage constraint a la Gertler and Kiyotaki (2010) into an endowment economy. The model features a calibrated financial sector, recursive preferences, and an independently and identically distributed consumption growth process. The leverage constraint makes intermediary equity capital (net worth) to be an important state variable that affects asset prices and helps to understand a wide variety of dynamic asset pricing phenomena. Rather than a log-linear approximation method, I use a global method that allows for occasionally binding constraint to solve the model, and show the global method is critical for quantifying asset pricing implications.

Quantitatively, an i.i.d. consumption growth shock, calibrated to match the standard deviation of the aggregate consumption growth, is amplified and accumulated through the propagation mechanism of the leverage constraint, and has large and long-lasting effects on asset prices, which are absent in the model without frictions. In particular, the model produces a high equity premium (in log units) of 4.1%, a significant share (78%) of the equity premium observed in the data, a low interbank interest rate volatility of 0.58%, consistent with the data (0.55%), and a persistent and procyclical variation of price-dividend ratio, with first order autocorrelation of 65%, relatively lower than that in the data (89%). The equity premium is strongly countercyclical in the model, and predictable with the leverage ratio of aggregate financial intermediary sector, a pattern I confirm in the data. The model also produces an average stock market volatility of 16.5%, only slightly lower than a volatility of 19.8% in the data.

The leverage constraint effectively introduces a wedge between interest rates on interbank and household loans. As a distinct implication from the model, the loan spread widens significantly in the credit crunch which features a large drop in intermediary net worth. This patten is consistent with the evidence that high TED spread[2] coincides with low price-dividend ratio and high stock market volatility, as shown in Figure 1.

I emphasize the importance of using a global method that accounts for occasionally binding constraint to solve the model. In the benchmark calibration with a moderate risk aversion of 10 and a calibrated financial sector, the global solution suggests the constraint only binds for about

[1] In this paper, the financial intermediary sector is meant to capture the entire banking sector, including commercial banks, investment banks as well as hedge funds. Thus, I use "financial intermediary sector" and "banking sector", "financial intermediaries" and "banks", interchangeably. For the composition of aggregate financial intermediary sector, see Table 1 in Appendix 7.3.

[2] TED spread is measured by the spread between 3-month LIBOR rate in U.S. dollars and 3-month U.S. government treasury bill rate.

15% of the time. A third order local approximation method, imposing the assumption that the constraint is always binding around the steady state, greatly exaggerates the volatilities of asset prices and equity premium.

There are two main ingredients in the model. First, I build a stylized leverage constraint faced by financial intermediary into an otherwise standard endowment economy. As in Gertler and Kiyotaki (2010), a limited enforcement argument that financial intermediary can divert a fraction of bank assets and default on deposits provides a microfoundation for the leverage constraint. In particular, the debt financing capacity to an intermediary is proportional to the equity capital of the intermediary times a leverage multiple. In this setup, the intermediary net worth strongly affects asset prices through an adverse dynamic feedback effect: a negative consumption shock lowers the intermediary net worth, increases the probability that constraint becomes binding in the future, and therefore reduces the borrowing capacity of the intermediary sector today and in the future. Lower borrowing capacity results in lower demand for risky assets. In the equilibrium, the intermediary sector still holds all the risky assets. To clear the market, the asset price has to fall, and risk premium has to rise. The resulting fall in asset price further lowers the net worth. An initial small i.i.d. consumption shock is endogenously amplified through this propagation mechanism.

The leverage constraint also opens up an endogenous channel of countercyclical equity premium and stock market volatility, even though consumption growth is homoscedastic. The equilibrium asset prices are more sensitive to the fundamental shocks when the intermediary net worth is low. As the financial intermediary sector becomes more financially constrained, both the exposure of market return to consumption shock (i.e. return beta) and the market price of the consumption shock increase, and thus contribute to a higher equity premium. In the model, price-dividend ratio and leverage ratio of the aggregate intermediary sector predict long-horizon equity returns. Both the slope coefficients and $R^2$ line up with the data relatively well at all horizons. And the model also captures the volatility feedback effect; that is, a consumption shock, as a negative innovation to market return, is a positive innovation to return volatility.

As a distinct feature of the model, the leverage constraint introduces a wedge between interest rates on interbank and household loans. This spread, as a measure of the tightness of leverage constraint, is countercyclical and widens significantly in bad times when the intermediary sector is extremely financially constrained. I posit a retail interbank market where the banks can trade Arrow-Debreu securities (in zero net supply) that pay one unit of net worth given a certain state among themselves frictionlessly (i.e. the bank cannot default on them), assuming the banks have monitoring technology in evaluating and monitoring their borrowers. Under this asset market

structure, the banks are unconstrained in choosing risky assets and interbank loans, though they are constrained agents to obtain debt from the household. The augmented stochastic discount factor suggested by the bank's portfolio choice problem price risky assets and interbank risk-free debt. It depends not only on household consumption, but also on intermediary equity capital. The banker dislikes assets with low return when aggregate consumption is low, and when his financial intermediary has low net worth. However, the interest rate on household loans is priced by a different stochastic discount factor, which is suggested by the household optimization problem. In a credit crunch, modeled as a large drop in intermediary net worth so that the constraint binds, the banks are strongly liquidity constrained to lend net worth to others, and therefore the market clearing condition drives up the interbank interest rate.

The second ingredient of the model is that quantitatively I rely on recursive preferences (Kreps and Porteus, 1978; Epstein and Zin, 1989) which allow for a separation between the intertemporal elasticity of substitution (IES, hereafter) and risk aversion, and consequently permit both parameters to be simultaneously larger than 1. I calibrate the recursive preference with a moderate risk aversion of 10 and an IES of 1.5, consistent with Bansal and Yaron (2004). In this economy, when the IES is larger than 1, the level of interest rate on household loans is low, consistent with the data. Furthermore, a high IES (larger than 1) is also critical to produce high equity premium. In the CRRA utility case, as the risk aversion increases, the IES, which is the reciprocal of risk aversion, decreases simultaneously, and leads the average leverage ratio of the financial intermediary sector to decrease very rapidly. This significantly lowers the volatility of the stochastic discount factor, due to lower volatility of shadow price of net worth. In contrast, when the IES is larger than 1, the average leverage ratio of the financial sector only decreases slowly with the risk aversion, therefore, maintains a volatile stochastic discount factor, and thus a high equity premium.

Computationally, I use a recursive method to construct a global solution which accounts for occasional binding constraint. The theoretical underpinnings of the recursive method are developed in a companion paper (Ai, Bansal and Li, 2012), while this paper focuses on the economics and quantitative analysis of the model. In the paper, I emphasize the importance of allowing for occasional binding constraint on quantifying the asset pricing implications. In the macroeconomics literature, equilibrium is often derived by log-linearizing around the steady state and assuming the constraint is always binding, for instance, Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2011), among others. As a result, this method does not allow me to study the model nonlinearity and off-steady-state dynamics, which are the key to internally generate the time-varying equity premium and stock volatility. Furthermore, even a higher order local approximation method imposing the assumption that the constraint is

always binding around the steady state is still problematic. In the parameter con guration with which the probability of a binding constraint is low, for instance, the benchmark calibration, a third order local approximation method that forces the constraint to be always binding around the steady state greatly exaggerates the volatilities of asset prices and equity premium. I use Den Haan and Marcet simulation accuracy test (1994) to con rm the advantage of the global method over a local approximation method.

My analysis contributes to several strands of literature. First, existing consumption based asset pricing models have been successful in specifying preferences and cash ow dynamics to explain a high and countercyclical equity premium in an endowment economy (Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Barro, 2006). However, these models allow no roles for nancial intermediary, but assume that a representative household is marginal in pricing all the assets, therefore, they cannot speak to the close relationship between nancial intermediary equity capital and aggregate stock market. They also shed no light on interest rate spread between interbank and household loans. In this paper, I show the single channel of a leverage constraint not only links asset prices to intermediary net worth, but also provides an additional important channel to understand a wide variety of asset market phenomena, even with an i.i.d. consumption growth process. The success of the model does not rely on a very high e ective risk aversion as in habit model, or on consumption risks beyond the business cycle frequency, for instance, long-run risks or rare diasters, which are hard to detect empirically in the data.

Second, this paper is directly related to Maggiori (2012) and He and Krishnamurthy (2012*b*) on nancial intermediary and asset pricing. As a continuous time adaptation of Gertler and Kiyotaki (2010) type of leverage constraint into an endowment economy, Maggiori (2012) is a special case of the model in this paper, in which the constraint never binds in the equilibrium. Thus, it has neither implications for interest rate spread, nor implications of occasionally binding constraint on asset pricing. In He and Krishnamurthy (2012*b*), the nancial intermediary faces an equity

icantly negative risk-free interest rate in the crisis, which suggest that model implied consumption volatility of marginal investor in the crisis state is very high, and thus induces a large precautionary saving e ect to lower the risk-free rate. Second, in my model, the ampli cation e ect is quantitatively large around the steady state where the constraint is not binding, due to the fact that the concern about potential future losses in net worth depresses the stock market today. However, He and Krishamurthy (2012*b*) framework is only to capture the risk premium behavior in crises, but features no ampli cation e ect in the unconstrained region.

Third, the paper also relates to the theoretical literature on intermediary frictions. There are two broad classes of theories: leverage-constraints theories and equity risk-capital constraints. Both theories start with the assumption that intermediaries are constrained in raising more equity. They share two common predictions: First, intermediary equity (or net worth) is the key state variable that a ects asset prices. Second, the e ect of intermediary equity on asset prices is nonlinear, with a larger e ect when the intermediary equity is low. The leverage-constraints models include Geanakoplos and Fostel (2008), Adrian and Shin (2010) and Brunnermeier and Pedersen (2009), Danielsson et al. (2011), Geanakoplos (2012), and Adrian and Boyarchenko (2012)*:* Gertler and Kiyotaki (2010) type of frictions lies in the rst category. He and Krishnamurthy (2012*a*) and Brunnemeier and Sannikov (2012) are examples of equity risk-capital models. The goal of this paper is di erent from the theoretical literature to propose alternative microfoundations for nancial frictions, rather I focus on the quantitative asset pricing implications of a stylized type of leverage constraint as in Gertler and Kiyotaki (2010), which has been widely studied in the macroeconomic and policy related literature.

More broadly, this paper is related to the literature in macroeconomics studying the e ects of nancial frictions on aggregate activity, including Kiyotaki and Moore (1997), Calstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrsit (1999), among others. These papers focus on the credit frictions faced by non- nancial borrowers. Gertler and Kiyotaki (2010) introduces a leverage constraint between household and nancial intermediary, also see Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2011), Gertler and Karadi (2012), among others. The equilibrium in these works is derived by log-linearizing around the steady state and assuming the constraint is always binding. Instead, I use a global method to solve the model, and emphasize that accounting for occasionally binding constraint is very important for quantifying asset pricing implications of the model. My work contributes to the literature by arguing that quantitative analysis on macroeconomic e ects and policy evaluations of nancial frictions should take into account the importance of occasionally binding constraint on asset price dynamics, which lie in the center of the propagation mechanism of nancial frictions.

The remainder of the paper is organized as follows: I present the model setup and de ne the competitive equilibrium in Section 2. In Section 3, I outline model solution, computation and discuss some analytical results in asset pricing. Section 4 presents benchmark model's performance in various aspects. Section 5 provides some additional asset pricing implications, and Section 6 concludes and lays down several extensions on my research agenda. Model derivations, data sources and computation details are provided in the Appendix.

# 2 The Model Setup

I embed a nancial intermediary sector with a leverage constraint a la Gertler and Kiyotaki (2010) into a standard Lucas (1978) endowment economy .

There are three sectors in the economy, namely, households, nancial intermediaries (banks), and non- nancial rms. I assume households cannot invest directly in the risky asset market by holding the equity of non- nancial rms. There is a limited market participation, also see Mankiw and Zeldes (1991), Basak and Cuoco (1998), or Vissing-Jorgensen (2002). Instead, households can only save through a risk-free deposit account with banks. Each household owns a unit mass of banks coming in overlapping generations. Banks borrow short-term debt from households[3], and invest in the equity of the rms. In addition to assisting in channeling funds from households to non- nancial rms, banks engage in maturity transformation. They hold long term assets and fund these assets with short term liabilities (beyond their own equity capital). In addition, the banking sector in this model is meant to capture the entire banking sector, including commercial banks, investment banks as well as hedge funds.

Time is discrete and in nite, $t = 0; 1; 2; \ldots$. The non- nancial rms in this economy are modeled as in a Lucas (1978) tree economy which pays aggregate output every period. The aggregate output is denoted by $Y_0; Y_1; Y_2; \ldots$. The log growth rate of the output process is given by

$$\log \frac{Y_{t+1}}{Y_t} = {}_y + \quad "_{y;t+1};$$

in which $"_{y;t+1}$ is an i.i.d. random variable with mean zero and unit variance, modeled as a nite-state Markov chain. The parameter captures the aggregate consumption volatility.

I use $Q_t$ to denote the price of the Lucas tree at period $t$, and thus the total return on the

[3]To motivate a limited enforcement argument later, it is best to think of banks only obtaining deposits from households who do not own them.

Lucas tree, $R_{y;t+1}$, is de ned as

$$R_{y;t+1} = \frac{Q_{t+1} + Y_{t+1}}{Q_t}:$$

## 2.1 Households

There is a unit mass of identical households who makes intertemporal consumption and saving decisions. I collapse all households into a single representative household. He is in nitely lived and maximizes the objective function,

$$\max_{fC_t;B_t g_{t=0}^{\infty}} E_0 \left[ \sum_{t=0} {}^t u(C_t) \right] ; \tag{1}$$

where $C_t$ is the period $t$ consumption. I consider a constant relative risk aversion (CRRA, hereafter) instantaneous utility function with risk aversion parameter , $u(C_t) = \frac{1}{1} C_t^1$ : In the subsequent quantitative analysis (Section 4), I use more general recursive preferences (Kreps and Porteus, 1978; Epstein and Zin, 1989), which disentangle the risk aversion with IES. This is quantitatively important for asset pricing implications as discussed in Section 4.1 and Section 4.5. More details about recursive preferences are provided in Appendix 7.1.

The household can only save through a risk-free deposit account with banks. Let $f \ {}_t g_{t=0}^{1}$ denote the stream of (stochastic) income that the household receives, and $R_{f;t}$ denote the one-period risk-free interest rate for a loan (made by the household to the banks) that pays o on date $t + 1$. A set of budget constraints (2) is described as the following:

$$\begin{aligned} C_0 + B_0 &= {}_0; \\ C_t + B_t &= B_{t \ 1} R_{f;t \ 1} + {}_t; \ t \quad 1: \end{aligned} \tag{2}$$

In the above formulation, the household receives a stream of income, $f \ {}_t g_{t=0}^{1}$ and makes his consumption and saving decisions. $C_t$ is the period t consumption choice, and $B_t$ is the amount he deposits in the one-period risk-free bond, which pays a gross interest rate $R_{f;t}$ in the next period. I will show later on, ${}_t$ is the amount of wealth transferred from the banking sector to the household at period $t$. That is, his ownership of the banks pays o over time as an income stream $f \ {}_t g_{t=0}^{1}$. Technically, the $f \ {}_t g_{t=0}^{1}$ sequence is constructed so that it can be easily veri ed that $C_t = Y_t$ satis es the budget constraint.

## 2.2 Financial Intermediaries

The banks come in overlapping generations. Denote $n^t_{t+j}$ to be the total amount of net worth held by all generation $t$ banks at period $t + j$, and $s^t_{t+j}$, the total number of shares in the Lucas tree held by all generation $t$ banks at period $t + j$. I use $_t$ to denote the Arrow-Debreu price of one unit of consumption good at period t denominated in terms of time 0 consumption goods. Under this notation, the price of a unit of consumption good at period $t + j$ denominated in terms of period $t$ consumption good is $\frac{\Lambda_{t+j}}{\Lambda_t}$. Given the price system $f \ _t g^{1}_{t=0}$, a generation $t$ bank maximizes the present value of its future cash ow by choose:f

$$\max_{f s^t_{t+j}; n^t_{t+j+1} g^{\infty}_{j=0}} E_t \left[ \sum_{j=1} \frac{t+j}{t} (1 \quad )^{j \ 1} \ n^t_{t+j} \ : \right] \tag{3}$$

In each period, a fraction of the bank is forced to liquidate, in which case, their net worth is paid o as dividend. The remaining fraction (1 ) will survive to the next period. The liquidation fraction/probability is i.i.d. across banks and time. As a result, the total fraction of a generation $t$ survived until period $t + j$ is (1 )$^{j \ 1}$, and a fraction is paid out as dividend. Note that the bank and household share the same stochastic discount factor, Gertler and Kiyotaki (2010) provide an \insurance story" to justify this.

Equation (4) is the initial condition of banks' net worth. The initial generation starts with initial net worth $N_0$. After that, in each period, the household uses a fraction of the Lucas tree to set up new banks, as assumed in Gertler and Kiyotaki (2010). Therefore, $[Q_t + Y_t]$ is the initial net worth of the generation $t$ bank at period $t$.

$$n^t_t = \quad [Q_t + Y_t] \ if \ t \quad 1; \ \ n^0_0 = N_0: \tag{4}$$

Equation (5) is the law of motion of net worth. At period $t + j$, the bank started with net worth $n^t_{t+j}$ and chooses hold $s^t_{t+j}$ shares of the stock. Each share pays $Q_{t+j+1} + Y_{t+j+1}$ in the next period, which is the rst term on the right hand side of (5). However, the bank has to borrow $s^t_{t+j} Q_{t+j} \quad n^t_{t+j}$ from the household in order to nance the purchase of the stock. The second term on the right hand side of (5) is the amount of loan repayment the bank has to deliver to the household in period $t + j + 1$.

$$n^t_{t+j+1} = s^t_{t+j} [Q_{t+j+1} + Y_{t+j+1}] \quad s^t_{t+j} Q_{t+j} \quad n^t_{t+j} \ R_{f;t+j}; \ for \ all \ j \quad 0: \tag{5}$$

Equation (6) is the participation constraint, motivated by a limited enforcement argument in Gertler and Kiyotaki (2010). At period $t + k$ in the future, the banker has an opportunity to divert a fraction of bank assets at its market price and default on its debt. And the depositors can only recover (1 ) fraction of bank asset, due to limited enforcement. Because the depositors recognize the bank's incentive to divert funds, they will restrict the amount they lend. In this way

is the total amount of net worth of all banks at date $t$ that comes from existing banks (banks of generation $t$ 1 and older): At period t-1 all existing banks together own one share of the Lucas tree, which pays o $Q_t + Y_t$. They have net worth $N_{t\ 1}$, and borrowed $Q_{t\ 1}$ $N_{t\ 1}$ to buy the tree. Consequently, $(Q_{t\ 1}$ $N_{t\ 1})\, R_{f;t\ 1}$ is the amount of interest they have to return to the household. The second part of the right-hand side of equation (10), $[Q_t + Y_t]$, is the amount of net worth that is newly injected into the banking sector at period $t$. Recall that each period the household use fraction of the Lucas tree to set up a new generation of banks.

The market clearing condition also include (12) and (13). Equation (12) implies the household and the bankers together own the Lucas tree. In particular, the household owns part of the Lucas tree directly, through $_0$, and owns part of the Lucas tree indirectly, through the banks, which is $N_0$. Equation (13) has the following interpretation: in period $t$, a fraction of all existing banks are forced to liquidate, and their net worth ows into the household. At the same time, the household also used fraction of the value of the Lucas tree to set up new banks. This completes the discussion of the market clearing conditions. Of course, given the budget constraint and market clearing conditions, one of (in each period) is redundant according to Walras' law.

I also need certain consistency condition:

$$_t = \frac{^t u'(C_t)}{u'(C_0)};$$

which captures the \insurance story" that Gertler and Kiyotaki (2010) tells.

# 3 Model Solution

In this section, I outline the main steps in deriving the solution, highlighting the economic mechanism linking intermediary equity capital and the asset prices. Detailed derivations are provided in the Appendix 7.2.

## 3.1 State Variable and its Dynamics

In this economy, nancial intermediary equity capital is an important state variable that a ects asset prices. As I comment below, in the discrete time setup, it turns out to be more convenient to use normalized debt, instead of net worth, as the state variable. Both debt and net worth measure the capitalization of nancial intermediary sector, and therefore, I may use them interchangeably in explaining the model intuitions.

I define normalized debt level as,

$$\mathbf{b}_t = \frac{B_{t-1} R_{f;t-1}}{Y_t};$$

as the state variable of the economy, where

$$B_{t-1} = Q_{t-1} \quad N_{t-1};$$

is the total amount of debt that the banks borrow from the household sector in period $t-1$. Because this is a growth economy, I normalize quantities and prices by total output, and denote:

$$q(\mathbf{b}_t) = \frac{Q(\mathbf{b}_t)}{Y_t}; \quad \frac{Y_{t+1}}{Y_t} = g_{t+1}; \quad \hat{n}_t = \frac{N_t}{Y_t}; \tag{14}$$

The law of motion for the state variable is therefore,

$$\mathbf{b}_{t+1} = \frac{R_{f;t}}{g_{t+1}} f(1 \quad) \mathbf{b}_t + ( \quad) q(\mathbf{b}_t) \quad (1 \quad + \quad) g: \tag{15}$$

One advantage of using $\mathbf{b}_t$ as the state variable is that: given today's $\mathbf{b}_t$ and an initial guess of the price functional $q(\,)$, the law of motion (15) determines $\mathbf{b}_{t+1}$ in close form. This property facilitates an iterative procedure to compute the equilibrium, as discussed in Section 3.4. However, if I use normalized net worth $\hat{n}_t$ as the state variable, a choice in Maggiori (2012) and He and Krishnamurthy (2012), I find that the law of motion of normalized net worth $\hat{n}_t$ in this discrete time context is not in closed form.

I can now express the current period net worth, $\mathbf{b}_t$, as a function of the state variable $\mathbf{b}_t$:

$$\begin{aligned} \mathbf{b}_t &= q(\mathbf{b}_t) \quad \frac{\mathbf{b}_{t+1} g_{t+1}}{R_{f;t}}; \\ &= (1 \quad + \quad) q(\mathbf{b}_t) \quad [(1 \quad) \mathbf{b}_t \quad (1 \quad + \quad)]: \end{aligned} \tag{16}$$

## 3.2 Recursive Formulation of Bank's Problem

I first set up some notations. I use $M_{t+1}$ to denote the one-period stochastic discount factor implied by household problem, as standard in the asset pricing literature. That is,

$$M_{t+1} = \frac{t+1}{t} = \frac{u'(C_{t+1})}{u'(C_t)}:$$

Euler equation implies:

$$E(M_{t+1})R_{f;t} = 1:$$

Both $M_{t+1}$ and $R_{f;t}$ do not depend on $\beta_t$, rather, they are determined by aggregate consumption growth process in the equilibrium. Note that with i.i.d. consumption growth, $R_{f;t}$ is a constant, which I denoted as $R_f$:

The bank's optimization problem has a recursive representation:

$$V(\beta_t; n_t) = \max_{\{s_t; n_{t+1}\}} E_t[M_{t+1} f \quad n_{t+1} + (1 \quad) V(\beta_{t+1}; n_{t+1})g] \tag{17}$$

$$subject\ to: n_{t+1} = s_t[Q(\beta_{t+1}) + Y_{t+1}] \quad [s_t Q(\beta_t) \quad n_t]R_f; \tag{18}$$

$$E_t[M_{t+1} f \quad n_{t+1} + (1 \quad) V(\beta_{t+1}; n_{t+1})g] \quad s_t Q(\beta_t): \tag{19}$$

Given initial wealth $n_t$ and the current state $\beta_t$, the bank chooses control variables $(s_t; n_{t+1})$, subject to the constraints. The constraint (18) essentially determines $n_{t+1}$ given the choice $s_t$ and the realization of the random variables exogenous to the maximization problem. I do not substitute out $n_{t+1}$ just to save notation. Since $n_{t+1}$ depends on $s_t$, constraint (19) restricts the choice of $s_t$.

I conjecture that $V(\beta_{t+1}; n_{t+1})$ is of the form[4]:

$$V(\beta_{t+1}; n_{t+1}) = \quad (\beta_{t+1}) n_{t+1}; \tag{20}$$

in which $(\beta_{t+1})$ is the shadow price of net worth at time $t+1$. In this case, the maximization problem can be written as:

$$V(\beta_t; n_t) = \max_{\{s_t; n_{t+1}\}} E_t[M_{t+1} f \quad + (1 \quad) \quad (\beta_{t+1})g\, n_{t+1}] \tag{21}$$

$$subject\ to: n_{t+1} = s_t[Q(\beta_{t+1}) + Y_{t+1}] \quad [s_t Q(\beta_t) \quad n_t]R_f;$$

$$E_t[M_{t+1} f \quad + (1 \quad) \quad (\beta_{t+1})g\, n_{t+1}] \quad s_t Q(\beta_t):$$

Given $f \quad (\beta_{t+1}); Q(\beta_{t+1})g$, I de ne

$$v(\beta_t) = \quad + (1 \quad) E_t[M_{t+1} \quad (\beta_{t+1})]R_f; \tag{22}$$

in which $v(\beta_t)$ is the shadow price of net worth at date $t$ if the participation constraint is not

[4]Note this is not saying that the equilibrium solution is nonlinear. It says, given equilibrium prices, the bank's value function is linear. The equilibrium prices are highly nonlinear, and are determined by some nonlinear method.

binding for any bank. Also, I de ne

$$P(\mathbf{b}_t) = \frac{E_t[M_{t+1} f \quad + (1 \qquad ) \quad (\mathbf{b}_{t+1}) g(Q(\mathbf{b}_{t+1}) + Y_{t+1})]}{v(\mathbf{b}_t)}. \tag{23}$$

in which $P(\mathbf{b}_t)$ is the equilibrium price of the Lucas tree in the case where the participation constraint does not bind for any bank. Note that $v(\mathbf{b}_t)$ and $P(\mathbf{b}_t)$ is completely determined once the functional form of $f \quad (\mathbf{b}_{t+1}); Q(\mathbf{b}_{t+1}) g$ is known.

As shown in the Appendix 7.2, I can summarize the equilibrium conditions with a compact notation.

$$Q(\mathbf{b}_t) = \frac{v(\mathbf{b}_t) P_t(\mathbf{b}_t) + v(\mathbf{b}_t) N_t(\mathbf{b}_t) \wedge \quad P_t(\mathbf{b}_t)}{v(\mathbf{b}_t) +}. \tag{24}$$

Also,

$$(\mathbf{b}_t) = v(\mathbf{b}_t) \_ \frac{Q(\mathbf{b}_t)}{N_t}. \tag{25}$$

in which $P(\mathbf{b}_t)$ and $v(\mathbf{b}_t)$ are given by (22) and (23). Here I used the short-hand notation $x \wedge y \quad \min\{x; y\}$ and $x \_ y = \max\{x; y\}$. Obviously, $Q(\mathbf{b}_t) \quad P(\mathbf{b}_t)$ and $(\mathbf{b}_t) \quad (\mathbf{b}_t)$, and strict inequality holds if and only if the participation constraint is binding.

## 3.3 Parameter Requirement

Parameter Assumption: I focus on the parameter that the lowest possible realization of consumption growth, $g_L$, is bounded by:

$$(1 \qquad) R_f < g_L < \frac{(1 \qquad) R_f}{(1 \qquad + \quad)}.$$

The rst part of the inequality implies that the minimum consumption growth rate of the economy cannot be too low. The intuition is that if the shocks are too low, a long enough sequence of bad shocks will send the total debt level in the banking sector to in nity, which cannot be consistent with any equilibrium. This observation has important consequences. For example, it implies that it would be inappropriate to consider a discrete time model with normal shocks, because the shocks are unbounded. The log-linearization method ignores this equilibrium restriction.[5]

In this economy, a fraction of net worth exits the banking sector and a fraction of the market value of the Lucas tree is injected back into the banking sector in each period. If is small

[5] This is not an issue in continuous time given Maggiori(2012)'s experiment, as in continuous time, as time interval shrinks, so does the size of the shocks.

enough, or is large enough, the bank will eventually get out of the constraint. The second part of the inequality makes sure that we focus on the interesting case that is large enough and is small enough, so that the economy will not grow out of the constraint with probability one.

The theoretical results on the parameter assumptions are provided in Ai, Bansal and Li (2012).

## 3.4 Computation

The literature[6] usually uses a local approximation method to solve the model with Gertler and Kiyotaki (2010) type of participation constraint, imposing the assumption that the constraint is always binding around the steady state. One exception is Maggiori (2012), which features a analytical global solution up to a system of ordinary di erential equations (ODEs) in a continuous time setting with log utility. In this paper, I use a global method to solve the model with recursive preferences in a discrete time context, allowing for occasionally binding constraint. In Section 4.1, I use quantitative experiments to show that the global method allowing for occasionally binding constraint is critical to quantify the asset pricing implications in such a model.

There are several reasons which make the model computation special. First, this model features an incomplete market, and thus the competitive equilibrium de ned in Section 2.3 does not correspond to a social planner's solution. Instead, we need to solve the competitive equilibrium directly. Second, because of the occasionally binding constraint (19), standard local approximation methods, for instance, perturbation method, cannot be used, unless we impose the assumption that the constraint always binds around the steady state. As such, I use a recursive method, the theoretical underpinnings of which are developed in Ai, Bansal and Li (2012), to construct the global solution. Third, because of the nonlinearity of the model and my focus on nonlinearity-sensitivity of asset prices with the state variable, I solve the model on a large number of grid points to ensure accuracy.

To summarize the intuition of an iterative procedure to solve the model, the following system (26), (27), (28), and (29) de nes a mapping $f(\mathbf{b}^0); q(\mathbf{b}^0)g = ) f(\mathbf{b}); q(\mathbf{b})g$, in which I use the convention that \0" denotes next period quantities. This system is normalized version of the system (22)-(25).

$$v(\mathbf{b}) = + (1 \quad) E[M^0 \quad (\mathbf{b}^0)] R_f; \tag{26}$$

$$p(\mathbf{b}) = \frac{E[M^0 f \quad + (1 \quad) \quad (\mathbf{b}^0)g f q(\mathbf{b}^0) + 1g g^0]}{v(\mathbf{b})}; \tag{27}$$

[6] An incomplete list includes Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2011); Gertler and Karadi (2012), among others.

me to sum across individual banks to obtain the relation for the demand for total bank assets as a function of total net worth,

$$Q_t \quad \overline{\phantom{x}}^{t} N_t:$$

Note that the demand for total bank assets is equal to $Q_t$, because all the rm equity is concentrated in the banking sector, and the total number of shares of equity is normalized to 1.

Second, the maximum leverage ratio depends on the aggregate state variable $\mathbf{b}_t$, and is countercyclical, as the shadow price of net worth $_t$ is high in bad times when net worth is scarce. This model feature is consistent with the empirical evidence on the leverage ratio of the aggregate intermediary sector, as shown in Figure 2.

Expecting that a bank will be able to abscond with stocks purchased with loans from household, household will require a collateral posted against the loans. Therefore, the participation constraint can be also rewritten/reinterpreted and aggregated as a collateral constraint, as follows:

$$B_t \quad \overline{\phantom{x}}^{\mathbf{t}} \quad 1 \quad N_t: \tag{30}$$

On the left hand side of (30), the aggregate loans from household sector, $B_t$, is equal to $Q_t \quad N_t$, as one of the market clearing conditions (11). The right hand of (30) is equal to aggregate net worth of the banking sector with a multiplier. It can be considered as the collateral required by the household to post against the loans.

### 3.5.2 Setup of the Asset Market

I posit a retail interbank market where the banks can trade Arrow-Debreu securities (in zero net supply) that pay one unit of net worth given a certain state among themselves. Suppose that the banks have a better enforcement/monitoring technology than households, therefore, the Arrow-Debreu securities are traded frictionless, i.e. no banks can default on them. Due to zero net supply, the market clearing condition pins down the Arrow-Debreu prices. In this sense, the stochastic discount factor suggested by the banks' portfolio choice problem (de ned in equation (32)) can price all the assets traded frictionlessly among banks, with their payo s being replicated by the Arrow-Debreu securities. Two classes of such assets of my interest are discussed in order.

First, risky assets. I distinguish between the unobservable return on a claim to aggregation output (consumption), $R_{y;t+1}$, and the observable return on the market portfolio, $R_{m;t+1}$; the latter is the return on the aggregate dividend claim. As in Campbell and Cochrane (1999) and Bansal and Yaron (2004), I model aggregate consumption and aggregate dividend as two separate

processes. In particular, the log growth rate of aggregate dividend is speci ed as:

$$\log \frac{D_{t+1}}{D_t} = {}_d + ' \, "_{y;t+1} + '_d \, "_{d;t+1}:$$

in which $"_{y;t+1}$ is the consumption shock speci ed as an i.i.d. random variable with nite state Markov chain as before, and $"_{d;t+1}$ is standard Normally distributed, and captures the dividend growth shock that is uncorrelated with consumption growth shock. Two additional parameters $' > 1$ and $'_d > 1$ allow me the calibrate the overall volatility of dividends (which is larger than that of consumption in the data) and its correlation with consumption. I use $Q_{d;t}$ to denote the price of the dividend claim, and the market return is thus de ned as,

$$R_{m;t+1} = \frac{Q_{d;t+1} + D_{t+1}}{Q_{d;t}}:$$

Second, the interbank loans that lend one unit of net worth today and return (pay back) $R^L_{f;t}$ units in the next period, which $R^L_{f;t}$ denotes the gross interest rate.

### 3.5.3 Asset Pricing

In this section, I discuss the equilibrium conditions that determines the returns of three kinds of assets, namely, interest rates on household and interbank loans, and the returns for risky assets.

The interest rate on household loans is determined by the Euler equation of household problem, una ected by frictions and has the standard interpretation of the optimal trade-o between consumption and savings.

Lemma 1 *The interest rate for the loans from the household sector, $R_{f;t}$, must satisfy*

$$E\,[M_{t+1}]\,R_{f;t} = 1:$$

Under the asset market structure in the interbank market discussed in last section, although the banks re constrained in obtaining household deposits, they are unconstrained in choosing risky assets and interbank loans. The stochastic discount factor suggested by the bank's portfolio choice problem price the risky assets and the interbank loans.

Lemma 2 *The returns, $R_{t+1}$; for any assets that nancial intermediary can trade frictionlessly among themselves (i.e. "frictionless" means that bank cannot default on them), including $R_{m;t+1}$; $R_{y;t+1}$ and $R^L_{f;t}$; must satisfy*

$$E \quad M_{t+1} \quad + (1 \quad ) \quad {}_{t+1} \quad R_{t+1} \quad = \quad {}_t; \tag{31}$$

*in which*

$$
\begin{aligned}
{}_t &= {}_t \frac{P_t}{Q_t}; \\
&= {}_t + \quad 1 \quad \frac{{}_t}{{}_t} :
\end{aligned}
$$

I use $\tilde{M}^f_{t+1}$ to denote the "augmented stochastic discount factor" implied by bank's optimization problem,

$$\tilde{M}^f_{t+1} = M_{t+1} \frac{+ (1 \quad ) \quad {}_{t+1}}{{}_t}; \tag{32}$$

which can price all the assets traded frictionlessly among banks. Beside $M_{t+1}$, the intertemporal marginal rate of substitution of consumption, $\tilde{M}^f_{t+1}$ also depends on an additional component, ${}_{t+1}$, which I de ne as:

$${}_{t+1} = \frac{+ (1 \quad ) \quad {}_{t+1}}{{}_t}. \tag{33}$$

The term, $+ (1 \quad ) \quad {}_{t+1}$; is a measure of shadow price of net worth at the next period, which is a weighted average of marginal value of net worth given the bank is forced to liquidate or not. Based on the equation (31), ${}_t$ can be interpreted as the (risk adjusted) present value (in term of consumption good) of investing one unit of net worth for one period, which is a measure of the marginal value of net worth at current period. Thus, we can think of the second component, ${}_{t+1}$;as the shadow price appreciation from period $t$ to $t+1$: And the augmented stochastic discount factor has the interpretation of the intertemporal marginal rate of substitution with respect to additional unit of net worth. $\tilde{M}^f_{t+1}$ depends not only on household consumption, but also on intermediary equity capital. The banker dislikes assets with low return when aggregate consumption is low, and when his nancial intermediary has low net worth/high debt.

Up to a log-normal approximation[8], I use $m$, and $r$ denote the logarithm terms, and derive a two-factor model for risk premium for all assets:

$$E_t \left[ r_{t+1} \quad r^L_{f;t} \right] + \frac{1}{2} var_t (r_{t+1}) = \quad cov_t (m_{t+1}; r_{t+1}) \quad cov_t \left( {}_{t+1}; r_{t+1} \right) : \tag{34}$$

One the right hand side of equation (34), the rst term, $cov_t (m_{t+1}; r_{t+1})$, is standard as in the economy without frictions. The second term, $cov_t \left( {}_{t+1}; r_{t+1} \right)$, is responsible for asset

[8]The log-normality assumption may not be a good approximation here, as the model endogenously generates negative skewness and excess kurtosis to asset prices. This assumption facilitates to obtain a two-factor asset pricing equation for expressional purpose. The model computation and qualitative results in the paper do not rely on this assumption.

pricing impacts for the additional channel of a leverage constraint. As shown in 4.3, the non-linear sensitivity of the marginal value of net worth, $_{t+1}$, with respect to a fundamental shock, translates into countercyclical exposure of $_{t+1}$ to the shock, and therefore, generates countercyclical market price of risk.

Note that two interest rates are priced by di erent stochastic discount factors, therefore, there is an interest rate spread, as stated in the following lemma:

Lemma 3 *The interest rate spread, de ned as the di erence between interest rate on interbank loans, $R^L_{f;t}$, and interest rate on household, $R_{f;t}$, is equal to zero when participation constraint is not binding, but becomes strictly positive when the constraint binds.*

First, we have $R_{f;t} = R^L_{f;t}$ whenever the constraint is not binding, because in this case, the leverage constraint is slack and both loans act as a perfect substitue. Second, we have $R_{f;t}$ $R^L_{f;t}$ when the intermediary sector is constrained. From the demand perspective, interbank borrowing is very attractive. It allows banks to invest in the stock without a ecting their debt capacity with the household. As a result, all banks want to borrow from each other on the interbank market. Market clearing requires interest rate to go up to clear the market. I will provide more intuitions on the interest rate spread in Section 4.3 through quantitative results.

# 4 Quantitative Results

In this section, I calibrate the model at an annual frequency and evaluate its ability to replicate key moments of both cash ow dynamics and asset returns. I focus on a long sample of U.S. annual data (1930 2011), including pre-war data, whenever the data is available. I begin with evaluating the model performance with CRRA utility, and compare the simulation accuracy between the global method and a third order local approximation method. Then, I focus on the benchmark model with recursive preferences, based on calibrated parameters reported in Table 2, and extensively discuss its quantitative asset pricing implications. Appendix 7.3 provides more details on the data sources.

## 4.1 Quantitative Evaluation the Solution Method

I begin with the model with CRRA utility and compare the performance of the global method used in this paper with a third order local approximation method. I argue that using a global method which allows for occasionally binding constraint is critical to quantify the asset pricing implications of nancial frictions.

First, I focus on CRRA utility case at different levels of risk aversion, namely, $\gamma = 1$ (log utility); $\gamma = 2$ and $\gamma = 5$, which are commonly used in the macroeconomics literature. For each calibration experiment, I keep all the other parameters the same as in the benchmark calibration, summarized in Table 2, and I compare the same model with the global method and a third order local approximation method implemented by the Dynare++ package. For each experiment, the moments from different solution methods are listed in two adjacent columns. The results are reported in Table 3.

I make the following observations. First, even with CRRA utility at low levels of risk aversion, for instance, $\gamma = 1$ (log utility); or $\gamma = 2$, the probability of constrained region is still low, around 20 30%. When risk aversion increases, the probability of constrained region rapidly decreases. Second, it is surprising but interesting to see that the model's implied equity premium decreases with risk aversion, and this pattern behaves in the opposite direction as compared with the standard Lucas economy without frictions. In CRRA utility case, the IES, as the reciprocal of the risk aversion, decreases with risk aversion, and leads the average leverage ratio to decrease dramatically, and in turn makes the volatility of shadow price of net worth to decrease rapidly. Since the dampening effect from the volatility of shadow price of net worth dominates the marginal rate of substitution of consumption, the first component in $\hat{M}_{t+1}$ as defined in equation (32), the augmented stochastic discount factor becomes less volatile and equity premium decreases. This experiment conveys the message that with CRRA utility, the financial frictions are not likely to have large asset pricing implications, because there is a strong trade-off between the contributions of two components in the augmented stochastic discount factor to the market price of risk. In Section 4.5, I will come back to this point and argue that when we incorporate recursive preferences with an IES larger than 1, the dampening effect discussed here is much weaker, and financial frictions generate significant impacts on asset prices.

It is also noteworthy that as the probability of constrained region decreases with risk aversion, the model's simulated moments suggested by the local approximation method have larger discrepancies with those of the global method. To further illustrate this point, in Table 4, I fix the risk aversion at $\gamma = 2$, and compare the model results for different bank asset divertible fractions $= 0.2; 0.4;$ and $0.8$. As above, for each experiment, I keep all the other parameters the same as in the benchmark calibration, summarized in Table 2. Since the parameter directly affects the incentive for banks to divert by increasing its outside option value, the probability of constrained region is monotonically increasing with : As suggested by the global solution, in the high case ( $= 0.8$), the constraint is almost always binding, while in the low case ( $= 0.2$); the prob(binding) is as low as $0.03$. Clearly, in the high case, the third order local approximation

solution performs very well, and reports very close moments to the global solution. However, in the low case in which the constraint rarely binds, the local approximation solution which imposes the assumption that the constraint always binds around steady state, greatly exaggerate the asset price volatilities, and therefore overstate the equity premium. In particular, in the low theta case ( = 0:2); the volatilities of price-dividend ratio and interbank interest rate are overestimated by more than twice and 10 times, respectively. And the equity premium is overestimated by more than 5 times.

I use the Den Haan and Marcet simulation accuracy test (1994) to compare the computation accuracy of the two solution methods. The basic idea is to construct the test statistic to measure the distance of simulated Euler equation error from zero. Under null hypothesis of exact numerical solution, the test statistic follows a $^2$ distribution. Additional details on constructing the test statistic are provided in Appendix 87. Figure 3 and 4 report the results for high case. In particular, they plot the empirical cumulative distribution of test statistic (based on 500 simulations of 1000 annual observations) versus its true $^2$ distribution under the null hypothesis for the global method and the local approximation method respectively. Both gures show that the empirical cumulative distributions are close to the true distribution under the null hypothesis. This implies that a third order local approximation method works well when *prob*(*binding*) is high. Figure 5 and 6 compare the results for low case. Figure 5 shows that the global method still works well, however, the local approximation method fails in the sense that the empirical cumulative distribution of simulation accuracy test statistic is far from its true distribution under the null hypothesis.

In sum, in order to quantify the asset pricing implications of nancial intermediary, we need to go to recursive preferences that which allow for a separation between the IES and risk aversion, and consequently permit both parameters to be simultaneously larger than 1, and use a global solution method which accounts for occasionally binding constraint.

## 4.2 Parameter Values

In this section, I discuss the parameter values in the benchmark calibration, which are summarized in Table 2.

Following Bansal and Yaron (2004), I set the relative rate of risk aversion, , to be 10, and the elasticity of intertemporal substitution, , to be 1:5. I set the discount factor, , to be 0:994 to match the level of risk-free interest rate for the household loans in the data.

In the log output growth process, the parameters $_y$ and are calibrated to match the mean and volatility of the consumption growth in the data. Similarly, $_d$ matches the average log

dividend growth rate. Two additional parameters in the log dividend growth process, $' > 1$ and $'_d > 1$ allow me to match the overall volatility of dividends (wTbi4c0(dividlarger(matc)2)1(han(mat

## 4.3 Basic Properties of the Model's Solution

In this section, I show the basic properties of the model's solution. In particular, I present the equilibrium prices, conditional volatilities of the market return and stochastic discount factor, and the equilibrium market return and risk-free interest rates, as functions of the state variable in this economy, i.e. the normalized debt level, $b$.

### 4.3.1 Equilibrium Prices

Figure 7 shows the equilibrium price-dividend ratio and marginal value of net worth as functions of normalized debt, $b$, of the banking sector.

I make the following observations: First, I assume the realized consumption growth is bounded and satisfies the parameter restrictions as discussed in Section 3.3. This is important, otherwise, the equilibrium may not exist as shown in Ai, Bansal and Li (2012). In other words, if we assume that shocks are conditionally (log) Normal as in typical RBC models, there will be no equilibrium although the log-linearization method in Gertler and Kiyotaki (2010) still produces a solution. As a result of that assumption, the equilibrium level of debt will always be bounded between $b_{MIN}$ and $b_{MAX}$.

Second, the top panel shows that the equilibrium price-dividend ratio is monotonically decreasing in $b$. As a comparison, the price-dividend ratio is a constant in the Lucas economy without frictions. The intermediary normalized debt level strongly affects asset prices through an adverse dynamic feedback: A negative fundamental shock causes the losses of net worth and the accumulation of more debt, lowers the borrowing capacity of the intermediary today and into the future, and thus lowers the investment in risky asset market and depresses the stock prices, which further lowers the net worth. Importantly, note that the price-dividend ratio is low even when the constraint is not binding. The possibility of a binding constraint in the future lower the bank's capacity to invest in the stock today, and consequently lowers the market price of the stock. This implies that the amplification effect on risk premium is in action even in the unconstrained region, although the magnitude is smaller than in the constrained region.

With similar intuitions, the bottom panel shows that the marginal value of net worth is monotonically increasing in $b$. Note that in the standard Lucas economy it is a constant, and equal to 1.

Furthermore, the dashed line in bold in Figure 7 depicts the the equilibrium prices in the constrained region. In the region where the constraint is binding, the price-dividend ratio decreases sharply and the marginal value of net worth increases sharply. This implies that the effects of intermediary debt on asset prices are non-linear and are especially large in bad times when

the intermediary debt is high. That is, when the intermediary sector is extremely financially constrained, a negative fundamental shock is amplified to have large effects.

### 4.3.2 Conditional Volatility of Returns

Figure 9 presents the conditional standard deviation of the market return (in log units) as a function of normalized debt level $b$. As the banking sector becomes more financially constrained, the conditional volatility of market return increases. Due to the nonlinear sensitivity of price-dividend ratio with respect to the intermediary debt level as shown in the top panel of Figure 7, the conditional volatility of market return increases more sharply when the banking sector is more levered. The increasing conditional volatility with the adversity of the state implies that the exposure of market return on the consumption shock (i.e. return beta) is increasing in bad times, which is one of the important channels to generate higher equity premium in bad times. As a comparison, the conditional variance of the return is constant in the Lucas economy without frictions since the price-dividend ratio is a constant.

The model endogenously produces several effects that have been emphasized in the empirical literature. First, the conditional variance in stock returns is persistent. The state variable, $b$, is persistent, and it translates into a persistent conditional variance of stock returns. Second, the model endogenously generates a "leverage effect", that is, a consumption shock, as a negative innovation to market return, is a positive innovation to return volatility. Third, the conditional volatility of stock returns is countercyclical, and is higher when the intermediary net worth is low.

### 4.3.3 Conditional Volatility of Stochastic Discount Factor

Figure 8 presents the conditional standard deviation of stochastic discount factor (in log units) as a function of normalized debt level $b$. The conditional volatility of the stochastic discount factor determines the maximal Sharpe ratio. As the banking sector becomes more financially constrained, the conditional variance of stochastic discount factor increases. As discussed in Section 3.5, the stochastic discount factor depends not only on the aggregate consumption, but also on the shadow price of net worth. The second component increases more sharply when the leverage of the intermediary sector is high as shown in the bottom panel of Figure 7 and translates into higher volatility of the stochastic discount factor. The increasing conditional volatility of the stochastic discount factor with the adversity of the state implies that the market price of consumption shock is increasing in bad times. This is an important channel for generating countercyclical equity premium. As a comparison, the conditional volatility of the stochastic discount factor is constant in the Lucas economy without frictions since the shadow price of net worth is a constant at 1, and

the consumption growth is homoscedastic.

### 4.3.4 Equity Premium

Figure 10 presents the expected market return on levered dividend claim and two risk-free interest rates, i.e. the interbank interest rate, and the interest rate on household loans, as functions of the normalized debt level.

I de ne the equity premium as the spread between expected market return and interbank interest rate, $E_t \left[ r_{m;t+1} \right] - r^L_{f;t}$, as it is determined by the covariance of the augmented stochastic discount factor $m_{t+1}$ and the market return $r_{m;t+1}$. I make the following two observations. First, the equity premium increases with intermediary sector's normalized debt level, $b$. Second, the behavior of increases in the equity premium is asymmetric, namely, it increases much faster in the constrained region than in the unconstrained region. Both observations are explained by the fact that the equilibrium asset prices are more sensitive to the fundamental shocks when the intermediary net worth is low. As the nancial intermediary sector becomes more nancially constrained, both the exposure of market return to consumption shock (i.e. return beta) and the market price of the shock increase, and thus contribute to a higher equity premium. And the equity premium increases faster when intermediary is extremely under-capitalized.

### 4.3.5 Interest Rate Spread

Figure 10 shows two interest rates as functions of the normalized debt level, $b$. The interest rate on household loans, $r_{f;t}$, is a constant, and does not depend on the state variable $b$, as stated in Lemma 1. The interest rate on interbank loans $r^L_{f;t}$ is identical to $r_{f;t}$ when the constraint does not bind. However, when the constraint binds, the interest rate spread, denoted as $r^L_{f;t} - r_{f;t}$, becomes strictly positive, and increases with the state variable $b$. This pattern is consistent with the empirical evidence that in bad times when the banking sector is under-capitalized, the TED spread spikes.

In order to understand the response of interbank interest rate $r^L_{f;t}$, it is important to focus on the conditional mean of stochastic discount factor (in log units), i.e. $\log E_t \left[ \exp\left( m_{t+1} + \epsilon_{t+1} \right) \right]$, which is equal to $r^L_{f;t}$ (up to a log-normal approximation):

$$\begin{aligned} & \log E_t \exp\left(m_{t+1} + \right)_{t+1} \qquad (35) \\ = \; & E_t(m_{t+1}) + \frac{1}{2} var_t(m_{t+1}) \\ & + E_t\left(_{t+1}\right) + \frac{1}{2} var_t\left(_{t+1}\right) + cov_t\left(m_{t+1}; _{t+1}\right) : \end{aligned}$$

In the i.i.d. consumption growth case, the first term $E_t(m_{t+1}) + \frac{1}{2} var_t(m_{t+1})$ is constant. Figure 11 plots a decomposition of the rest two terms in the conditional mean of stochastic discount factor, i.e. $E_t\left(_{t+1}\right)$ and $\frac{1}{2} var_t\left(_{t+1}\right) + cov_t\left(m_{t+1}; _{t+1}\right)$ : Clearly, there are two forces determining the response of the interbank interest rate. First, the top panel shows that $E_t\left(_{t+1}\right)$ is decreasing in $b$: In the bad state with a negative shock which leads to a higher debt level, the net worth becomes more valuable today than the next period. Thus, the banks are very reluctant to lend net worth to others, instead they have strong incentive to borrow net worth and invest. Due to zero net supply, the market clearing condition drives up the interbank interest rate. Second, the bottom panel shows the second moment component $\frac{1}{2} var_t\left(_{t+1}\right) + cov_t\left(m_{t+1}; _{t+1}\right)$ increases in $b$. The precautionary savings effect decreases the interbank interest rate. As shown by the magnitude of two panels, the first effect dominates the precautionary savings effect, and thus overall the interest rate on interbank loans increases in response to a negative fundamental shock, when the constraint is binding.

## 4.4 The Performance of Benchmark Model

I repeatedly simulate 1000 artificial samples from the model, each with 81 annual observations. For each data moment, I report the median value, 2.5; 5; 95, and 97.5 percentiles, as well as the population value from a very long simulation (a long simulation of 10000 annual observations). The results are summarized in Table 5.

Designed by the calibration procedure, the model matches the aggregate consumption and dividend dynamics very well. It is noteworthy that by choosing two parameters, $'$ and $'_d$; i.e. the loadings of aggregate dividend growth on consumption growth shock and its own shock, the model roughly matches the correlation between consumption and dividend growth, and the overall volatility of dividend process.

I use two asset pricing moments, namely, the leverage ratio and the volatility of interest rate spread to calibrate the model. Not surprisingly, the model matches these two moments very well.

The model also performs very well in matching other asset pricing moments which are not

targeted in the calibration. First, the model internally generates a persistent uctuations of price-dividend ratio with rst autocorrelation of 65%, even though the driving consumption growth process is i.i.d. Note that in the Lucas economy without frictions, price-dividend ratio is a constant. In this economy, intermediary's debt level is a state variable that a ects asset prices, and thus price-dividend ratio inherits its positive serial correlation.

Second, the model produces a high equity premium (in log units) of 4:1%, a signi cant share (78%) of the equity premium observed in the data, and a stock market volatility of 16:5%, only slightly lower than a volatility of 19:8% in the data.

However, we also notice that there are some discrepancies between the model implied moments with the data. The model implied average interest rate spread is 0:15%, lower than 0:64% in the data. As I argued in Section 4.2, the model predicts zero interest rate spread when constraint does not bind, however, in the data, the TED spread is largely always positive even when the banking sector is well-capitalized. What's more, we only have TED spread for a short sample (1986 2011), therefore, the average spread may be driven high due to the inclusion of the recent nancial crisis period when the TED spread was enormously high. Another discrepancy is that the model understates the volatility of the log price-dividend ratio. In the model, the standard deviation of the log price-dividend ratio is 0:12, as compared with 0:45 in the annual data. Historical stock prices display low-frequency variation relative to cash ow, which is not captured in the model. The historical standard deviation of log price-dividend ratio is this high in part because stock prices were persistently high at the end of the sample period. In Bansal and Yaron (2004), the sample period ends at 1998, they obtain a lower standard deviation of 0:29 in the data, but still somewhat higher than in the model here.

Overall speaking, Table 5 suggests that the model performs relatively well to match both cash ow dynamics and asset pricing moments for U.S. data, given the driving force is an i.i.d. process. I could introduce a predictable component in expected consumption and dividend growth to further improve the persistence and standard volatility of price-dividend ratio.

## 4.5 Comparative Statics

To shed more light on the economic mepa ucialt dividhd26(somewhat)-327(h)1(9)1(n)27(ev)28(er,)r2aesas14r

being perturbed, all the other parameters are kept the same as in the benchmark calibration. In Table 7, all moments are reported from a very long simulation of data from the model at the annual frequency. The first column corresponding to the benchmark calibration as reported in Table 7.

### 4.5.1 Different Risk Aversion and Consumption Volatility

The first two variations consider changes in the risk aversion and the consumption volatility , the moments of which are reported in the second and third column, respectively.

Relative to the benchmark calibration of = 10; setting = 5 decreases the equity premium and the probability of entering the constrained region. When lower risk aversion, the intermediary sector is less conservative, and is willing to take a more risky portfolio, i.e. it has a higher average leverage ratio. Hence, the same consumption volatility is translated into a greater volatility of net worth, and the economy is more likely to hit a binding constraint state. This is a risk-taking effect. There is also a general equilibrium effect reinforcing the risk-taking effect. Due to a lower

price of risk falls and the banking sector is compensated less per unit of risk, and hence it has a lower average net worth level, which in turn leads to the constraint to bind more frequently.

### 4.5.3 Different Liquidation/Exit Probability

In the fifth column of Table 7, I increase $\zeta$; the fraction of banks forced to liquidate each period, from 0.12 to 0.16. This implies that the average survival duration decreases from 8.33 years to 6.25 years. As we can see from the Table 7, since every period there is a larger fraction of net worth paid back to the household sector, the banking sector tends to be more financially constrained, and have a higher average leverage ratio. Following the same "risk taking" story as stated above, higher risky position is translated into a higher volatility of the net worth and a higher equity premium. Higher volatility of the net worth leads the economy to enter the constrained region more often. This effect is also reinforced by the lower average net worth of the banking sector.

It is noteworthy that this experiment also reflects a amplification and persistence trade-off. With a higher $\zeta$, that is, a larger fraction of aggregation net worth paid back to the household sector each period, the equilibrium premium increases, however, the price-dividend ratio is less persistence, translated by a less persistent net worth process. This case is expected to feature a less return predictability.

### 4.5.4 Different Bank Asset Divertible Fraction

In the experiment shown in the last column, I increase the parameter $\theta$, which dictates the fraction of bank asset divertible, from 0.4 to 0.6. This mainly affects the average leverage ratio of the banking sector. As the banking sector can divert a larger fraction of bank assets, the leverage constraint allows a much lower average leverage ratio. As a result, the volatilities of net worth and of shadow price of net worth decrease, which leads to a decrease in equity premium and stock market volatility. Despite of lower average leverage, the probability of constrained region is still larger than in the benchmark case. This is because the right hand side threshold of the constraint increases, which makes it to bind more frequently.

### 4.5.5 Conditional Moments

Table 6 shows the model implied moments conditional on the leverage constraint being binding or not. Each panel of the table corresponds to a comparative statics experiment discussed above. As shown in the table, for all cases, the leverage ratio, Sharpe ratio and interest rate spread conditional on the constraint being binding is higher than those moments in the unconstrained

region.

# 5 Additional Asset Pricing Implications

## 5.1 Variance Decomposition of Price-Dividend Ratio

In this section, I replicate the variance decomposition of price-dividend ratio as in Cochrane (1992) and Campbell and Cochrane (1999). Table 8 presents the estimation results. Consistent with previous research, the estimates in the data find that more than 100 percent of the price-dividend ratio variance is attributed to expected return variation. A high price-dividend ratio signals a decline in subsequent real dividends, so it must signal a large decline in expected returns. The model is consistent with this feature in the data. Almost all (over 90%) the variation in price-dividend ratio is due to changing expected returns. This evidence repeats the intuition discussed above: the expected dividend growth is our model is constant over time, however, a negative fundamental shock, which causes the loss of net worth (or the accumulation of net debt), provides an endogenous channel of a discount rate shock, that greatly and persistently lowers the expected return.

An interesting point of comparison for my result is to the habit model in Cochrane and Campbell (1999). In that model, they modify the utility function of a representative investor to exhibit time-varying risk aversion, and therefore a negative fundamental shock is a discount rate shock by construction. Differently, I work on CRRA utility and recursive preferences as a more general utility function to disentangle risk aversion with IES, but generate an endogenous channel of time-varying equity premium as a function of the frictions in the economy.

## 5.2 Return Predictability

In this section, I provide the valuation on model's ability to endogenously generate return predictability. The left panel of Table 9 reports the results on predictability of mutli-period excess returns by the log price-dividend ratio. Consistent with evidence in earlier papers, In the data, the $R^2$ rises with maturity, from 4% at one year horizon to about 31% at the five year horizon. The model-implied predictability of equity return is somewhat lower. The slope coefficients in the multi-horizon return projections implied by the model are of the right sign and magnitude compared to those in the data.

The right panel of Table 9 shows evidence on predictability of multi-period excess returns by the log leverage ratio of the aggregate financial intermediary sector. In the data, the $R^2$ rises with

maturity, from 9% at one year horizon to about 28% at the ve year horizon. The model-implied predictability of equity return is comparable to those in the data, and the slope coe cients in the multi-horizon return projections implied by the model are of the right sign as those in the data. In sum, the empirical evidence presented in this section shows that the leverage constraint channel endogenously generates signi cant variation in equity premium.

## 5.3 Correlation Structure of Leverage Ratio

The economic mechanism in the model has strong implications for the correlation of leverage ratio with various asset market moments. In Table 10, I reports the correlations of leverage growth with price-dividend ratio, excess stock return, stock market integrated volatility and nancial asset growth of the intermediary sector.

In the literature, there are some discussions about the cyclicality of leverage ratio. In particular, Adrian and Shin (2010) documents that the leverage ratio of security broker-dealers is highly procyclical, by showing that leverage ratio of this particular type of nancial intermediary, constructed from Flows and Fund Table in U.S., is positively correlated with its asset growth. He, Khang and Krishnamurthy (2010) shows that there is large heterogeneity among di erent types of nancial intermediary. In particular, they document that in the period of 2007q1 to 2009q1, the broker-dealers shed assets, consistent with Adrian and Shin (2010)$^{0}$s evidence, however, the commercial banking sector increased asset holdings over this period signi cantly, and therefore, increased its leverage ratio. In this paper, the model intermediary sector is meant to capture the entire nancial intermediary sector. Thus, I follow the de nition in Adrian Moench and Shin (2011) to construct the leverage ratio of aggregate intermediary sector, with a coverage consistent with the model. The details about data construction are shown in the Appendix 7.3. I nd, in the data, the leverage growth of aggregate intermediary sector is negatively correlated with its asset growth, which suggests the leverage ratio is countercyclical. This is consistent with the model.

The data also suggests that in bad times when leverage ratio increases, stock price is low, the stock return decreases in the contemporaneous period, and stock market volatility increases. The benchmark model ts these correlation patterns in the data well.

## 5.4 Correlation Structure of Interest Rate Spread

As a distinct prediction, the model draws strong implications for the correlation of interest rate spread between interbank and household loans with price-dividend ratio, price-earnings ratio and the stock market volatility. Figure 1 shows the periods of signi cant widening of TED spread

coincide with those of dramatic increases in stock market volatility, and large decreases in price-dividend and price-earning ratios. In Table 11, I con rm these correlations. As I discussed in Section 4.3, the model is consistent with the correlation patterns in the data well. In the model, the interest rate spread, as a measure of the tightness of the credit constraint, spikes when the intermediary sector are extremely nancial constrained. The banks are constrained, and do not have liquidity to lend out to others, thus, the market clearing drives up the interest rate. On the other hand, low intermediary net worth depresses the stock market, and increases the stock market volatility, as we discussed above. These model predictions explain the empirical evidence very well.

## 5.5 Backward Looking Regression

In this section, I follow Bansal, Kiku and Yaron (2012) to evaluate the model by examining the link between price-dividend ratio and consumption growth. I replicate their empirical procedure and run the following regression:

$$p_{t+1} \quad d_{t+1} = \quad _0 + \sum_{j=1}^{X} \quad _j \quad c_{t+1 \quad j} + u_{t+1}:$$

In the actual data and in the simulated data, I regress the log of price dividend ratio on $L$ lags ($L = 1; 2; :::; 5$) of consumption growth. In the data, at all lag-lengths, predictability of the price dividend ratio by lagged consumption growth is close to zero. However, in the model, price dividend ratio predictability by lagged consumption has an $R^2$ of 42%. This is not surprising as prices in this model are driven primarily by the net worth, and hence, by movements in the lagged consumption, and a reduction in growth rates causes the loss in net worth, and thus increases the equity premium, and provide an endogenous positive discount shock, leading to a fall in current price-dividend ratio. This feature of the model is similar to the habit model in Campbell and Cochrane (1999). Both models are backward looking, in the sense that backward consumption plays an important role in determining current prices. The empirical evidence presented in this section proposes a challenge for asset pricing models with nancial intermediary.

# 6 Conclusion

In this study, I show nancial frictions are important for understanding a wide variety of dynamic asset pricing phenomena. I build a nancial intermediary sector with a leverage constraint a la

Gertler and Kiyotaki (2010) into a standard endowment economy with recursive preferences and an independently and identically distributed consumption growth process. Quantitatively, the

# 7 Appendix

## 7.1 Derivations of Equilibrium Conditions from Household Problem

In the benchmark model, the representative household is making optimal consumption and saving decisions by maximizing recursive preference (Kreps and Porteus, 1978; Epstein and Zin, 1989):

$$U_t = \left[ (1 - \beta) C_t^{\frac{1-\rho}{\theta}} + \beta \left( E_t \left[ U_{t+1}^{1-\rho} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\rho}};$$

subject to the budget constraint:

$$C_t + B_t = B_{t-1} R_{f;t-1} + \omega_t.$$

The Euler equation gives:

$$E_t [M_{t+1}] R_{f;t} = 1;$$

in which the stochastic discount factor is:

$$M_{t+1} = \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{\left( E_t \left[ U_{t+1}^{1-\rho} \right] \right)^{\frac{1}{1-\rho}}} \right)^{\frac{1}{\psi}-\rho}.$$

## 7.2 Derivations of Equilibrium Conditions from Bank's Problem

Based on the recursive representation of a typical individual bank's optimization problem as stated in (21).

Use the law of the motion to substitute out $n_{t+1}$. Let $\mu(\beta_t)$ denote the Lagrangian multiplier with respect to the participation constraint.

The first order condition with respect to $s_{t+1}$ is:

$$(1 + \mu(\beta_t)) E_t [M_{t+1} \{ \phi + (1 - \phi) \beta(\beta_{t+1}) \} \{ Q(\beta_{t+1}) + Y_{t+1} - Q(\beta_t) R_{f;t} \}] = \mu(\beta_t) Q(\beta_t). \tag{36}$$

The envelope condition with respect to $n_t$ is:

$$\mu(\beta_t) = (1 + \mu(\beta_t)) E_t [M_{t+1} \{ \phi + (1 - \phi) \beta(\beta_{t+1}) \}] R_{f;t}. \tag{37}$$

The complementary slackness conditions are:

$$\begin{aligned} \mu(b_t)\,[\ \ \lambda(b_t)\,n_t \quad\ s_t Q(b_t)] \ &= \ 0; \\ \mu(b_t) \quad\ &0; \end{aligned} \tag{38}$$

$$\lambda(b_t)\,n_t \quad\ s_t Q(b_t) \quad 0: \tag{39}$$

Since all the individual banks make the same decision, it allow us to have equilibrium conditions at the aggregation level. Equations (36), and (37) stay the same, and the complementary slackness conditions become:

$$\begin{aligned} \mu(b_t)\,[\ \ \lambda(b_t)\,N_t \quad\ Q(b_t)] \ &= \ 0; \\ \mu(b_t) \quad\ &0; \end{aligned} \tag{40}$$

$$\lambda(b_t)\,N_t \quad\ Q(b_t) \quad 0: \tag{41}$$

Given $f\ \ (b_{t+1}); Q(b_{t+1})g$, I de ne

$$v(b_t) = \quad + (1 \quad\ )\,E_t[M_{t+1}\ \ \lambda(b_{t+1})]\,R_{f;t}; \tag{42}$$

$v(b_t)$ is the shadow price of net worth at date $t$ if the constraint is not binding for any bank[10]. Also, de ne

$$P(b_t) = \frac{E_t[M_{t+1}\,f \quad + (1 \quad\ )\ \ \lambda(b_{t+1})g\,(Q(b_{t+1}) + Y_{t+1})]}{v(b_t)}: \tag{43}$$

$P(b_t)$ is the equilibrium price of the Lucas tree in the case where the participation constraint does not bind for any bank. Note that $v(b_t)$ and $P(b_t)$ are completely determined once the functional form of $f\ \ (b_{t+1}); Q(b_{t+1})g$ is known. (The prices $M_{t+1}$ and $R_f$ are trivially determined because it is an endowment economy.)

It is easy to show that the Lagrangian multiplier $\mu(b_t)$ can be expressed as

$$\mu(b_t) = \frac{\lambda(b_t)}{v(b_t)} \quad 1: \tag{44}$$

Use this relationship to substitute out $\mu(b_t)$, it is easy to show that the equilibrium conditions

[10]Note, here I adopt the following mathematical de nition of a "binding" constraint. "Binding" means the Lagrangian multiplier must be strictly positive. It rules out the case where the constraint holds with equality but the Lagrangian multiplier is zero.

are summarized by the following lammas.

Lemma 4 *(Equilibrium Price of the Lucas Tree)*

*Given the equilibrium pricing functional* $fQ(b_{t+1}); \quad (b_{t+1})g$*, we consider the equilibrium pricing functional* $Q(b_t)$

1. *Suppose*

$$v(b_t) N_t \qquad P(b_t); \tag{45}$$

*then in equilibrium, we must have:*

$Q(b_t) = P(b_t)$*, where* $P(b_t)$ *is given in (43).*

*The constraint (39) is not binding for any bank in the sense that the Lagrangian multiplier on the constraint must be* 0*.*

2. *Suppose*

$$0 < v(b_t) N_t < \quad P_t(b_t); \tag{46}$$

*then in equilibrium, we must have:*

*The price of the Lucas tree,* $Q(b_t)$*, satisfies:*

$$Q(b_t) = \frac{v(b_t)[P(b_t) + N_t]}{+ v(b_t)} < P_t(b_t): \tag{47}$$

*The constraint (39) is binding for all banks in the sense that the Lagrangian multiplier on the constraint must be strictly positive.*

3. *If* $N_t \quad 0$*, then equilibrium cannot exist.*

The three cases discussed above provide a complete characterization of the equilibrium at state $b_t$ given the price and quantities at state $b_{t+1}$. The first part of the lemma says that if the total net worth of the banking sector is large enough, then the participation constraint will not bind, and the equilibrium price of the Lucas tree is given by (43). Note, however, even if the constraint does not bind at time $t$, the price is still different from that in a frictionless Lucas model. This is because the possibility of a binding constraint in the future will affect today's price.

The second part of the lemma implies that if the total net worth is positive, but small, then the participation constraint will bind, and the equilibrium price has to drop (relative the price $P$)

to lower the outside value of the bankers. The third part of the condition says total net worth can never be zero or negative in equilibrium.

Given the above lemma, we can derive the functional form of $V(b_t; n_t)$. It is straightforward to show that if $V(b_{t+1}; n_{t+1})$ is linear in $n_{t+1}$ as in (20), then $V(b_t; n_t) = \quad (b_t)\, n_t$, and $\quad (b_t)$ is given by the following lemma.

Lemma 5 *(Equilibrium Value Function of the Financial Intermediary)*

*Given the equilibrium pricing functional* $f Q(b_{t+1}); \quad (b_{t+1})g$*, we consider the equilibrium pricing functional* $\quad (b_t)$*:*

1. *Under condition (45),*

$$(b_t) = v(b_t):$$

2. *Under condition (46),*

$$(b_t) = v(b_t) \quad \frac{fP(b_t) + N(b_t)g}{N(b_t)[\quad + v(b_t)]}: \tag{48}$$

To summarize the above two lemmas, under condition (45), the constraint does not bind, and $f \quad (b_t); Q(b_t)g$ can be constructed recursively from $f \quad (b_{t+1}); Q_{t+1}(b_{t+1})g$:

$$(b_t) = \quad + (1 \qquad) E_t[M_{t+1} \quad (b_{t+1})] R_{f;t}; \tag{49}$$

and

$$Q_t(b_t) = \frac{E_t[M_{t+1} f \quad + (1 \qquad) \quad (b_{t+1})g\, fQ(b_{t+1}) + Y_{t+1}g]}{(b_t)}: \tag{50}$$

Note that

$$\frac{E_t[M_{t+1} f \quad + (1 \qquad) \quad (b_{t+1})g]}{(b_t)} = \frac{1}{R_{f;t}}:$$

Note that on the right hand side of equations (49) and (50), all quantities are known except $f \quad (b_{t+1}); Q(b_{t+1})g$. So the system (49) and (50) de nes a mapping

$$f \quad (b_t); Q(b_t)g = T\, f \quad (b_{t+1}); Q(b_{t+1})g:$$

Under condition (46), I similarly de ne the mapping $f \quad (b_{t+1}); Q(b_{t+1})g = ) \quad f \quad (b_t); Q(b_t)g$. To save notation, we can summarize the two case with a compact notation. Using (48),

$$Q(b_t) = \frac{v(b_t) P(b_t) + v(b_t) N(b_t) \wedge \quad P(b_t)}{v(b_t) +}: \tag{51}$$

Also,

$$(b_t) = \quad (b_t) \_ \frac{Q(b_t)}{N_t}: \tag{52}$$

Here I used the short-hand notation $x \wedge y \quad \min\{x; y\}$ and $x \_ y = \max\{x; y\}$. Obviously, $Q_t(b_t) \quad P(b_t)$ and $(b_t) \quad (b_t)$, and strict inequality holds if and only if (46) is true, in which case the participation constraint is binding.

## 7.3 Data Sources

Consumption: Per capita consumption data are from the National Income and Product Accounts (NIPA) annual data reported by the Bureau of Economic Analysis (BEA). The data are constructed as the sum of consumption expenditures on nondurable goods and services (Table 1.1.5, lines 5 and 6) de ated by corresponding price de ators (Table 1.1.9, lines 5 and 6).

Dividend: The dividend process is constructed from VWRETD and VWRETX, i.e. the value weighted return on NYSE/AMEX including and excluding dividends, taken from CRSP. The construction of price-dividend ratio follows the data appendix in Bansal, Khatchatrian and Yaron (2005).

Earnings: Corporate earnings data are from corporate pro ts (earnings) after tax (in billions of dollars) from National Income and Product Accounts (NIPA) data reported by the Bureau of Economic Analysis (BEA) (Table 1.14, line 29). The construction of price-earnings ratio follows the data appendix in Bansal, Khatchatrian and Yaron (2005).

Market Return: Nominal market return is the value weighted return on NYSE/AMEX including dividends taken from CRSP. The real market return is computed by de ating the nominal return by corresponding price de ators (Table 1.1.9, lines 5 and 6).

Risk-free Rate: The nominal risk-free rate is measured by the annual 3-month T-Bill return. The real risk-free rate is computed by subtracting the nominal risk-free rate by expected in ation, a procedure detailed in Beeler and Campbell (2012).

TED Spread: Computed by the di erence between annualized 3-month LIBOR rate and 3-month T-bill rate. Both series are from FRED dataset.

Leverage Ratio: I follow Adrian, Moench and Shin (2011)'s composition of the aggregate nancial intermediary sector. From Flow of Funds Table in U.S. I aggregate the assets and liabilities of each component, and then compute the aggregate leverage ratio based on:

$$\text{Leverage}_t = \frac{\text{Aggregate Financial Assets}_t}{\text{Aggregate Financial Assets}_t \quad \text{Aggregate Liabilities}_t}$$

Integrated Volatility: Integrated variance is the sum of squared daily stock returns on NYSE/AMEX. Integrated volatility is the square root of integrated variance. The daily value weighted return data on NYSE/AMEX including dividends are taken from CRSP.

Table 1: Composition of Aggregate Financial Intermediary Sector

| Symbol | | Descriptions |
|---|---|---|
| FINBANK | | Banks |
| | CBSI | Charted depository institutions, excluding credit unions |
| | CU | Credit unions |
| FINPI | | Pension Funds and Insurances |
| | PCIC | Property-casualty insurance companies |
| | LIC | Life insurance companies |
| | PPF* | Private pension funds |
| | SLGERF* | State & local government employee retirement funds |
| | FGRF* | Federal government retirement funds |
| FINMF | | Mutual Funds |
| | MMMF* | Money market mutual funds |
| | MF* | Mutual funds |
| | CEF* | Closed-end funds and exchange-traded funds |
| SHADBANK | | Shadow Banks |
| | MORTPOOL* | Agency- and GSE-backed mortgage pools |
| | ABS | Issuers of asset-backed securities |
| | FINCO | Finance companies |
| | FUNDCORP | Funding corporations |
| SBRDLR | | Security brokers and dealers |

Notes - This Table is based on the de nitions in Adrian, Moench and Shin (2010). The component intermediaries denoted by *" means they are only nanced by equity.

## 7.4 Additional Details of the Numerical Solutions

I approximate the i.i.d. consumption shock $\varepsilon_{y,t}$ by a finite-state Markov chain. I x 5 realizations evenly spaced on the bounded interval [ 2 ; 2 ], in which denotes the consumption

Following Den Haan and Marcet (1994), the accuracy test consists of obtaining long simulations of the process and calculating

$$W_T = \frac{\sum_{t=1}^{T} \overline{w}_{t+1} \quad \overline{h}_t}{T};$$

where $\overline{w}_t$ and $\overline{h}_t$

Table 3: CRRA Utility: Di erent RRA

| | Data | Model | | | | | |
|---|---|---|---|---|---|---|---|
| | | = 1 | | = 2 | | = 5 | |
| | | Global | Local | Global | Local | Global | Local |
| Avg.Leverage | 3.67 | 4.38 | 2.72 | 3.76 | 3.72 | 2.51 | 2.60 |
| $E[log(\hat{\ })]$ | - | 2.49 | 2.70 | 2.02 | 2.02 | 1.61 | 1.58 |
| $E(r_m \quad r_f^L)$ | 4.58 | 1.75 | 1.92 | 1.09 | 1.40 | 0.99 | 1.57 |
| $E(r_f^L \quad r_f)$ | 0.64 | 0.44 | -0.05 | 0.56 | 0.37 | 0.12 | -0.48 |
| $[log(\hat{\ })]$ | - | 0.30 | 0.44 | 0.23 | 0.26 | 0.114 | 0.16 |
| $(p \quad d)]$ | 0.45 | 0.07 | 0.07 | 0.05 | 0.06 | 0.02 | 0.05 |
| $(r_m)$ | 19.79 | 17.34 | 17.55 | 16.15 | 16.82 | 14.06 | 15.24 |
| $(r_f^L)$ | 0.55 | 0.98 | 2.51 | 1.09 | 1.86 | 0.49 | 1.49 |
| $prob(binding)$ | | 0.28 | | 0.35 | | 0.11 | |

Notes - This table presents selected moments implied by the model with CRRA utility at di erent risk aversion parameters. Other parameters are kept the same as in the benchmark calibration in Table 2. All the moments reported are computed from a very long sample of simulated data. In columns \global", the moments are based on the global solution. In columns \Local", the moments are based on a third order local approximation method implemented using dynare++ package. Means and volatilities of returns and growth rates are expressed in percentage terms.

Table 4: CRRA Utility: Di erent Bank Assets Divertible Fraction

| | Data | Model | | | | | |
|---|---|---|---|---|---|---|---|
| = 2 | | = 0:2 | | = 0:4 | | = 0:8 | |
| | | Global | Local | Global | Local | Global | Local |
| $E[log(\hat{n})]$ | - | 1.96 | 3.16 | 2.02 | 2.02 | 2.08 | 2.08 |
| $E(r_m \quad r_f^L)$ | 4.58 | 0.96 | 5.54 | 1.09 | 1.40 | 0.44 | 0.35 |
| $E(r_f^L \quad r_f)$ | 0.64 | 0.05 | -3.85 | 0.56 | 0.37 | 2.30 | 2.32 |
| $[log(\hat{n})]$ | - | 0.31 | 1.39 | 0.23 | 0.26 | 0.11 | 0.11 |
| $(p \quad d)$ | 0.45 | 0.06 | 0.12 | 0.05 | 0.06 | 0.03 | 0.03 |
| $(r_m)$ | 19.79 | 16.21 | 23.65 | 16.15 | 16.82 | 14.67 | 14.55 |
| $(r_f^L)$ | 0.55 | 0.36 | 5.77 | 1.09 | 1.86 | 0.71 | 0.71 |
| $prob(binding)$ | | 0.03 | | 0.35 | | 1.00 | |

Notes - This table presents selected moments implied by the model with CRRA utility of risk aversion parameter of 2, at di erent fractions of bank assets divertible, . Other parameters are kept the same as in the benchmark calibration in Table 2. All the moments reported are computed from a very long sample of simulated data. In columns \global", the moments are based on the global solution. In columns \Local", the moments are based on a third order local approximation method implemented using dynare++ package. Means and volatilities of returns and growth rates are expressed in percentage terms.

Table 5: Dynamics of Growth Rates and Prices Based on Benchmark Calibration

| | Data | Benchmark Model | | | | | |
|---|---|---|---|---|---|---|---|
| | Estimate | Median | 2.50% | 5% | 95% | 97.50% | Pop |
| $E(\ c)$ | 1.83 | 1.78 | 1.30 | 1.39 | 2.18 | 2.30 | 1.80 |
| $(\ c)$ | 2.19 | 2.18 | 1.92 | 1.96 | 2.41 | 2.44 | 2.22 |
| $E(\ d)$ | 1.08 | 1.09 | -1.22 | -0.80 | 3.02 | 3.39 | 1.27 |
| $(\ d)$ | 10.98 | 10.90 | 9.27 | 9.69 | 12.35 | 12.54 | 10.93 |
| $corr(\ c;\ d)$ | 0.56 | 0.60 | 0.45 | 0.47 | 0.71 | 0.72 | 0.59 |
| $avg{:}leverage$ | 3.67 | 4.00 | 3.61 | 3.66 | 4.51 | 4.69 | 4.02 |
| $(leverage)$ | 1.65 | 0.93 | 0.50 | 0.54 | 2.32 | 2.78 | 1.22 |
| $E(r_m \quad r_f)$ | 5.22 | 4.04 | 1.25 | 1.60 | 6.62 | 7.06 | 4.07 |
| $E(r_m \quad r_f^L)$ | 4.58 | 3.86 | 1.01 | 1.31 | 6.57 | 6.96 | 3.90 |
| $(r_m)$ | 19.79 | 16.54 | 14.00 | 14.42 | 18.75 | 19.41 | 16.69 |
| $E(p \quad d)$ | 3.38 | 3.12 | 2.88 | 2.92 | 3.32 | 3.36 | 3.12 |
| $(p \quad d)$ | 0.45 | 0.12 | 0.08 | 0.08 | 0.15 | 0.16 | 0.12 |
| $AC1(p \quad d)$ | 0.86 | 0.62 | 0.43 | 0.47 | 0.75 | 0.77 | 0.65 |
| $E(r_f^L \quad r_f)$ | 0.64 | 0.15 | 0.03 | 0.03 | 0.40 | 0.45 | 0.17 |
| $(r_f^L)$ | 0.55 | 0.52 | 0.13 | 0.18 | 0.94 | 1.09 | 0.58 |

Notes - This table presents descriptive statistics for aggregate consumption growth, dividends, prices, the interest rate spread (i.e. the spread between interest rates for interbank and household loans). The data are real, sampled at an annual frequency and cover the sample period from 1930 to 2011, whenever the data are available. The sample period for leverage ratio is from 1945 to 2011. The sample period for interbank interest rate is from 1986 to 2011. The \Model" panel presents the corresponding moments implied by the model. The rst ve columns in the right panel represent percentiles of nite sample Monte-Carlo distributions. Population values (Pop) are computed from a very long sample of simulated data. Means and volatilities of returns and growth rates are expressed in percentage terms.

Table 6: Model Implied Conditional Moments

| | Unconstrained | Constrained |
|---|---|---|
| Panel A: Benchmark | | |
| Probability | 0.86 | 0.14 |
| Leverage Ratio | 3.69 | 6.02 |
| Sharpe Ratio | 0.21 | 0.46 |
| Interest Rate Spread | 0.00 | 1.21 |
| Panel B: = 5 | | |
| Probability | 0.77 | 0.23 |
| Leverage Ratio | 3.78 | 6.04 |
| Sharpe Ratio | 0.12 | 0.35 |
| Interest Rate Spread | 0.00 | 1.47 |
| Panel C: = 0:0156 | | |
| Probability | 0.58 | 0.42 |
| Leverage Ratio | 3.76 | 4.86 |
| Sharpe Ratio | 0.16 | 0.32 |
| Interest Rate Spread | 0.00 | 1.25 |
| Panel D: = 0:16 | | |
| Probability | 0.72 | 0.28 |
| Leverage Ratio | 4.22 | 6.34 |
| Sharpe Ratio | 0.23 | 0.45 |
| Interest Rate Spread | 0.00 | 1.55 |
| Panel E: = 0:6 | | |
| Probability | 0.33 | 0.67 |
| Leverage Ratio | 2.92 | 3.65 |
| Sharpe Ratio | 0.19 | 0.31 |
| Interest Rate Spread | 0.00 | 1.34 |

Notes - This table presents selected moments implied by the model conditional on being in the unconstrained versus constrained regions. Each panel corresponds to a comparative statics experiment in Table 7. All the moments reported are computed from a very long sample of simulated data. Means and volatilities of returns and growth rates are expressed in percentage terms.

Table 7: Comparative Statics

| | Benchmark | = 5 | = 0:0156 | IES = 0.5 | = 0.16 | = 0.6 |
|---|---|---|---|---|---|---|
| avg.leverage | 4.02 | 4.30 | 4.23 | 3.49 | 4.81 | 3.41 |
| $E[log(\hat{n})]$ | 2.86 | 2.82 | 2.88 | 2.15 | 2.38 | 2.69 |
| $[log(\hat{n})]$ | 0.25 | 0.28 | 0.19 | 0.20 | 0.26 | 0.19 |
| $AC1(p \quad d)$ | 0.65 | 0.65 | 0.63 | 0.70 | 0.55 | 0.67 |
| ( ) | 0.47 | 0.53 | 0.29 | 0.23 | 0.56 | 0.33 |
| ( ) | 0.14 | 0.16 | 0.11 | 0.10 | 0.17 | 0.10 |
| $E(r_m \quad r_f)$ | 4.07 | 2.96 | 2.64 | 3.33 | 4.83 | 4.19 |
| $E(r_m \quad r_f^L)$ | 3.90 | 2.62 | 2.11 | 3.03 | 4.39 | 3.29 |
| $(r_m)$ | 16.69 | 16.96 | 11.56 | 15.67 | 16.47 | 15.78 |
| $E(r_f^L \quad r_f)$ | 0.17 | 0.34 | 0.53 | 0.30 | 0.43 | 0.90 |
| $(r_f^L)$ | 0.58 | 0.87 | 0.93 | 0.78 | 0.98 | 1.01 |
| $prob(binding)$ | 0.14 | 0.23 | 0.42 | 0.23 | 0.28 | 0.67 |
| $amp:eff:$ | 2.68 | 3.61 | 2.89 | 2.09 | 3.03 | 2.26 |

Notes - This table presents selected moments implied by the model for comparative statics experiments. The rst column reports the moments based on benchmark calibration. Each of the rest 5 columns report the moments by changing one parameter, while keeping all the other parameters the same as in the benchmark calibration. All the moments reported are computed from a very long sample of simulated data. Means and volatilities of returns and growth rates are expressed in percentage terms. ( ) denotes the volatility of shadow value of net worth. ( ) denotes the volatility of $log(\ )$, de ned in equation (33).

Table 8: Variance Decomposition of Price-Dividend Ratio

| Source | Data | S.E. | Model |
|---|---|---|---|
| Dividends | -6% | (31%) | 2% |
| Returns | 108% | (42%) | 90.45% |

Notes - This table reports the percentage of $var(p \quad d)$ accounted for by returns and dividend growth rates:

$$100 \sum_{j=1}^{1} {}^{j} \frac{cov_t (p_t \quad d_t; x_{t+j})}{var_t (p_t \quad d_t)}$$

$x = \quad r$ and $\quad d$, respectively, and $\quad = \frac{1}{1+E(r)}$: The \model" column is based on a very long simulation of annual observations from the model with benchmark calibration.

Table 9: Return Predictability

| Predictor | p-d | | | log leverage | | |
|---|---|---|---|---|---|---|
| | Data | (S.E.) | Model | Data | (S.E.) | Model |
| $B(1)$ | -0.09 | (0.07) | -0.27 | 0.09 | (0.05) | 0.05 |
| $B(3)$ | -0.27 | (0.16) | -0.43 | 0.22 | (0.09) | 0.09 |
| $B(5)$ | -0.43 | (0.21) | -0.66 | 0.28 | (0.11) | 0.10 |
| $R^2(1)$ | 0.04 | (0.04) | 0.05 | 0.02 | (0.02) | 0.06 |
| $R^2(3)$ | 0.19 | (0.13) | 0.09 | 0.09 | (0.04) | 0.10 |
| $R^2(5)$ | 0.31 | (0.15) | 0.15 | 0.11 | (0.05) | 0.16 |

Notes - This table provides evidence on predictability of future excess return by log price-dividend ratio, and log leverage ratio of the aggregate intermediary sector. The entries correspond to regressing

$$r^e_{t+1} + r^e_{t+2} + ::: + r^e_{t+j} = \quad (j) + B(j)x_t + v_{t+j}$$

where $r^e_{t+1}$ is the excess return, j denotes the forecast horizon in years. $x_t$ denotes log price-dividend ratio for the left panel, and denotes log leverage ratio for the right panel. The entries for the model are based on 1000 simulations each with 81 annual observations. Standard errors are Newey-West corrected using 10 lags.

Table 10: Correlations of Aggregate Leverage Ratio and Asset Prices

| | Data | S.E. | Model |
|---|---|---|---|
| $corr(lev; p \quad d)$ | -0.71 | 0.20 | -0.44 |
| $corr(lev; r_m \quad r_f^L)$ | -0.75 | 0.19 | -0.93 |
| $corr(lev; IV)$ | 0.38 | 0.16 | - |
| $corr(lev; asset \quad growth)$ | -0.60 | 0.16 | - |

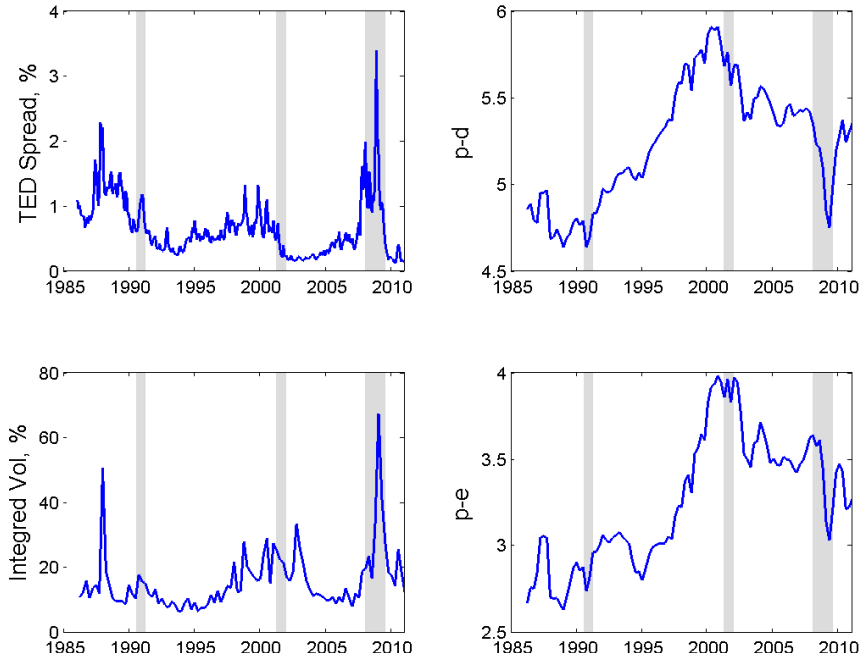
Notes - This table shows the correlations between log leverage growth of aggregate intermediary sector with asset market moments, including price-dividend ratio, stock excess return, stock market integrated volatility and nancial asset growth in the aggregate intermediary sector. The data are sampled at the annual frequency, ranging from 1945 to 2011. Data constructions are described in the Appendix 7.3. The numbers reported in \S.E." column are based on GMM Newey-West standard errors. The corresponding model implied correlations are reported whenever applicable, based on a very long sample of simulated data.

Table 11: Correlations of Interest Rate Spread and Asset Prices

| | Data | S.E. | Model |
|---|---|---|---|
| $corr(r_f^L \quad r_f; \quad lev)$ | 0.11 | 0.06 | 0.46 |
| $corr(r_f^L \quad r_f; p \quad d)$ | -0.42 | 0.20 | -0.77 |
| $corr(r_f^L \quad r_f; IV)$ | 0.32 | 0.15 | 0.40 |

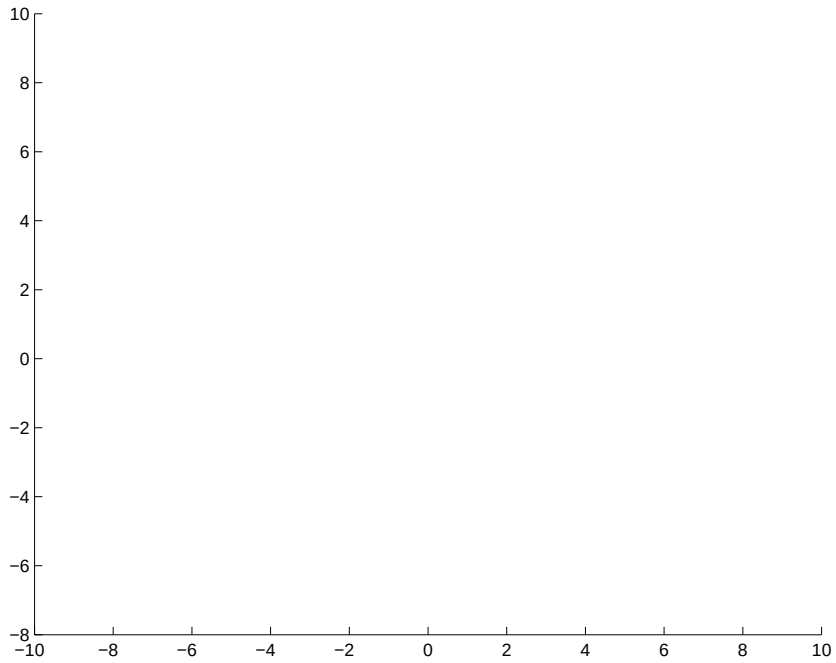
Notes - This table shows the correlations between TED spread with asset market moments, including log leverage growth of the intermediary sector, log price-dividend ratio, log price-earnings ratio and stock market integrated volatility. The data are sampled at the annual frequency, ranging from 1986 to 2011. Data constructions are described in the Appendix 7.3. The numbers reported in \S.E." column are based on GMM Newey-West standard errors. The corresponding model implied correlations are reported whenever applicable, based on a very long sample of simulated data.

Fig. 1: TED Spread, p-d ratio, p-e ratio and Integrated Volatility



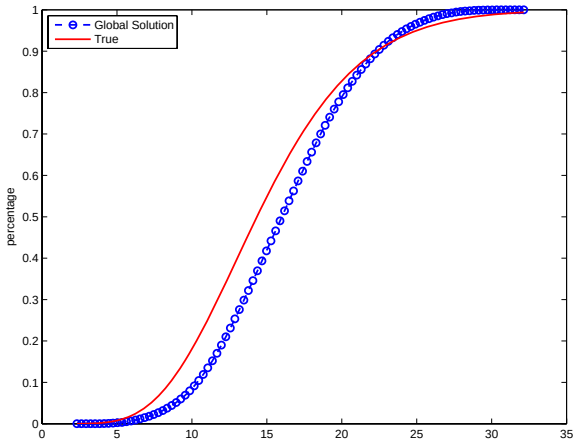

This gure plots TED spread, log p-d ratio, log p-e ratio and integrated volatility over the sample period 1986 to 2011. TED spread and integrated volatility are in annualized percentage. Shaded areas refer to NBER dated recessions. Data constructions are described in Appendix 7.3.

## Fig. 2: Leverage Ratio of Aggregate Financial Intermediary Sector



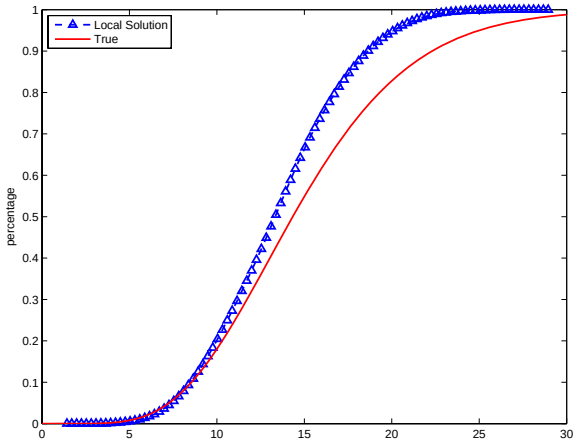

This figure shows scatter plots of the growth rate of financial assets (horizontal axis) versus the growth rate of leverage ratio (vertical axis) of the aggregate financial intermediary sector. The sample is at quarterly frequency, ranging from 1952q2 to 2011q4. Both axes are measured in percentage. The constructions of the total financial assets and leverage ratio of the aggregate financial intermediary sector are described in Appendix 7.3.

Fig. 3: Accuracy of Global Method (High Case)



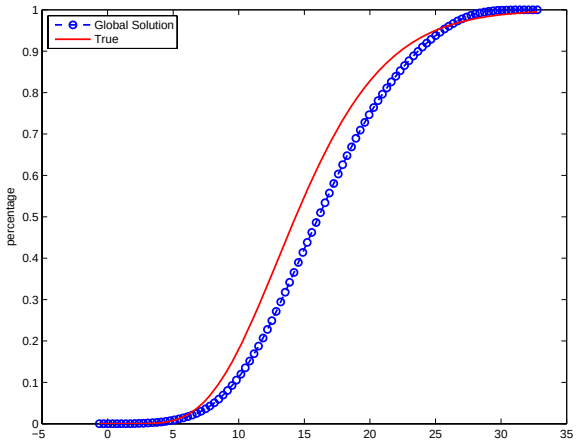

This gure shows the cumulative distribution function of the simulation accuracy test statistics suggested by Den Haan and Marcet (1994) and the corresponding $^2$ distribution under the null hypothesis. The realizations of the test statistics are based on 500 simulation paths, each with 1000 annual observations. The simulations are based on the global solution with high case ( = 0:8).

Fig. 4: Accuracy of Local Approximation Method (High Case)



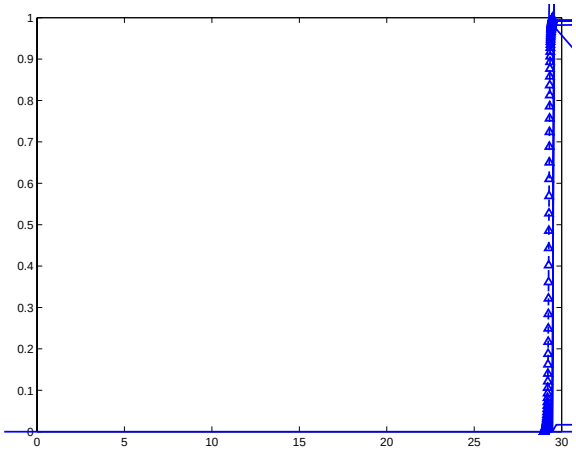

This gure shows the cumulative distribution function of the simulation accuracy test statistics suggested by Den Haan and Marcet (1994) and the corresponding $^2$ distribution under the null hypothesis. The realizations of the test statistics are based on 500 simulation paths, each with 1000 annual observations. The simulations are based on the third order local approximation solution with high case ( = 0:8).

Fig. 5: Accuracy of Global Method (Low Case)



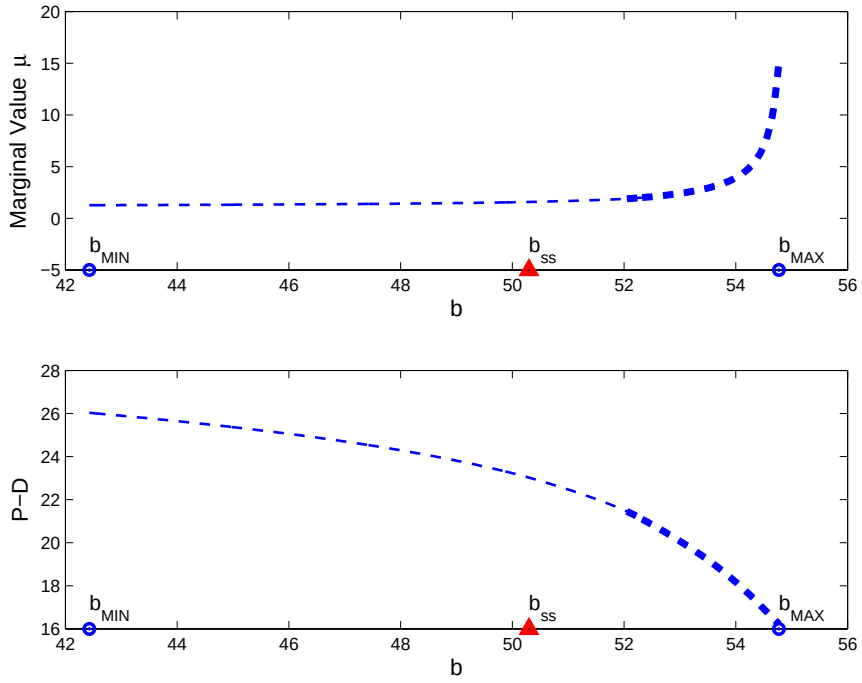

This gure shows the cumulative distribution function of the simulation accuracy test statistics suggested by Den Haan and Marcet (1994) and the corresponding $^2$ distribution under the null hypothesis. The realizations of the test statistics are based on 500 simulation paths, each with 1000 annual observations. The simulations are based on the global solution with low case ( = 0:2).

Fig. 6: Accuracy of Local Approximation Method (Low Case)



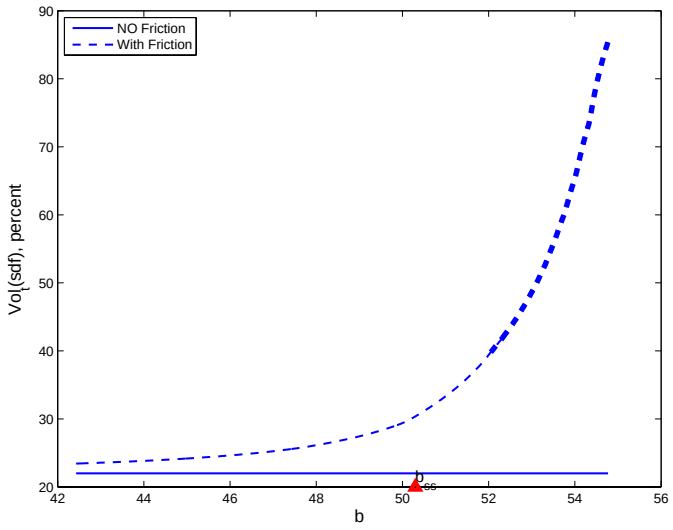

This gure shows the cumulative distribution function of the simulation accuracy test statistic suggested by Den Haan and Marcet (1994) and the corresponding $^2$ distribution under the null hypothesis. The realizations of the test statistics are based on 500 simulation paths, each with 1000 annual observations. The simulations are based on the third order local approximation solution with low case ( = 0:2).

Fig. 7: Equilibrium Prices as Functions of Normalized Debt Level, b



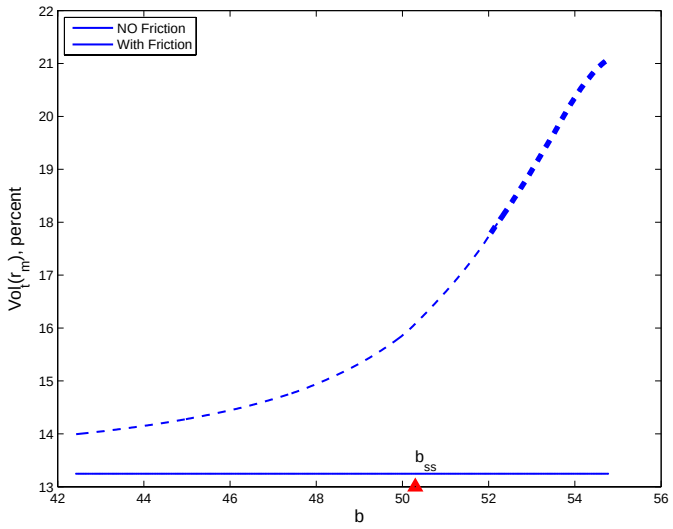

This gure shows the p-d ratio on aggregate dividend claim and the shadow price of net worth as functions of the state variable $b$. $b_{MIN}$ and $b$

FIG. 8 - Conditional Volatility of Log SDF as a Function of Normalized Debt Level, b



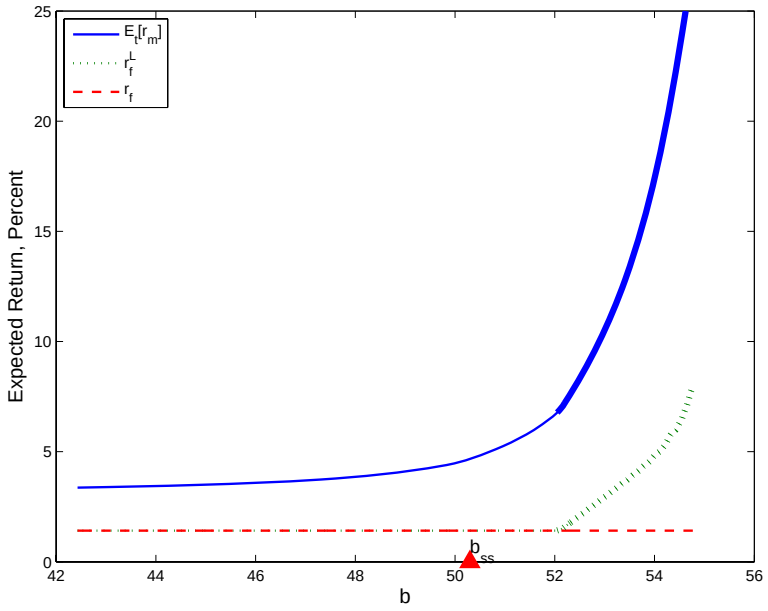

This figure shows conditional volatilities of stochastic discount factor with and without frictions as functions of the state variable $b$. $b_{SS}$ denotes the average debt level in this economy, suggested by a long simulation from the model. The part of curves highlighted in bold denotes the region at which the constraint is binding. The vertical axis is measured in annualized percentage. The parameters are based on the benchmark calibration summarized in Table 2.

Fig. 9: Conditional Volatility of Return as a Function of Normalized Debt Level, b



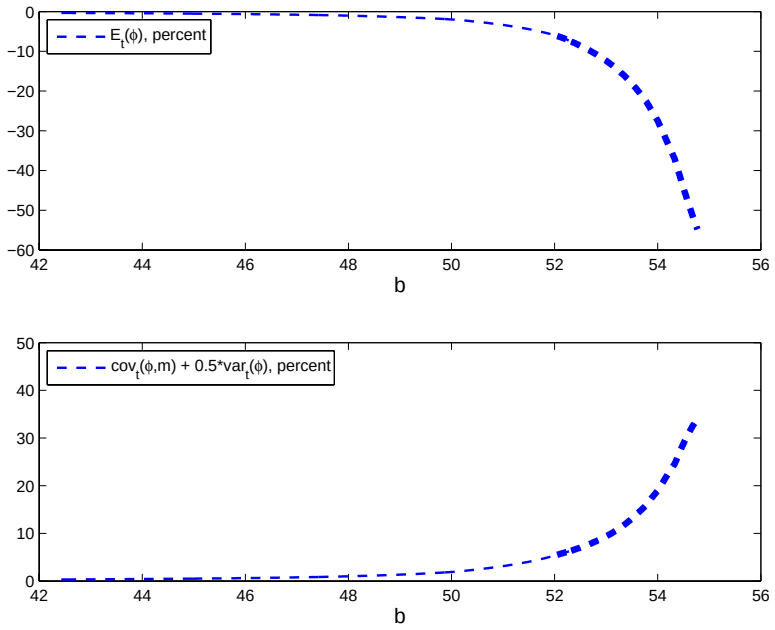

This figure shows conditional volatilities of market return with and without frictions as functions of the state variable $b$. $b_{SS}$ denotes the average debt level in this economy, suggested by a long simulation from the model. The part of curves highlighted in bold denote the region at which the constraint is binding. The vertical axis is measured in annualized percentage. The parameters are based on the benchmark calibration summarized in Table 2.

Fig. 10: Expected Returns as Functions of Normalized Debt Level, b



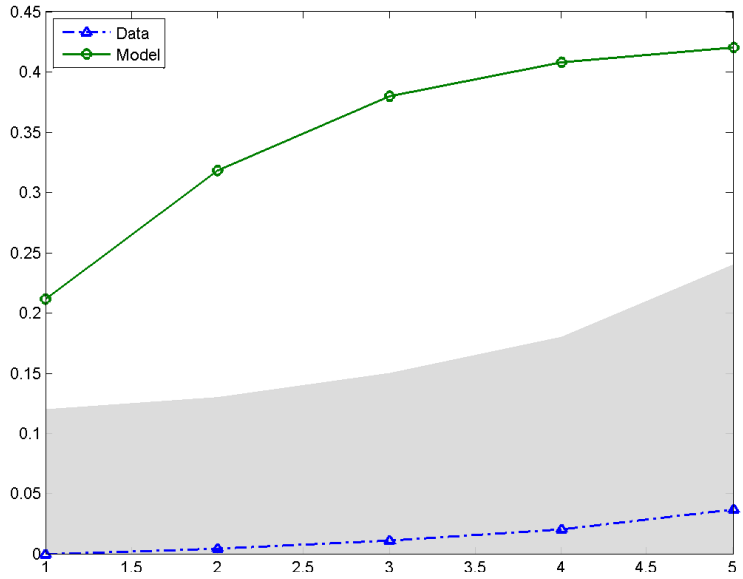

This figure shows the expected market return, interbank interest rate, and the interest rate on household loans as functions of the state variable $b$. $b_{SS}$ denotes the average debt level in this economy, suggested by a long simulation from the model. The part of curves highlighted in bold denotes the region at which the constraint is binding. The vertical axis is measured in annualized percentage. The parameters are based on the benchmark calibration summarized in Table 2.

Fig. 11: Decomposition of Stochastic Discount Factor




This gure shows the $E_t \quad _{t+1}$ and $\frac{1}{2}var_t \quad _{t+1} + cov_t \quad m_{t+1}; \quad _{t+1}$ , two components in the decomposition of the conditional mean of augmented stochastic discount factor, as shown in equation (35). $b_{ss}$ denotes the average debt level in this economy, suggested by a long simulation from the model. The part of curves highlighted in bold denotes the region at which the constraint is binding. The vertical axis is measured in annualized percentage. The parameters are based on the benchmark calibration summarized in Table 2.

Fig. 12: Price-dividend Ratio and Backward Consumption Growth




This figure plots the $R^2$ for regressing future log price-dividend ratio onto distributed lags of consumption growth:

$$p_{t+1} \quad d_{t+1} = \quad _0 + \sum_{j=1}^{K} \quad _j \quad c_{t+1} \quad _j + u_{t+1}$$

where $L$, the number of lags, is depicted on the horizontal-axis. The shaded area in the figure corresponds to the 95% confidence band in which data-based standard errors are constructed using a block-bootstrap. The data employed in the estimation are real, compounded continuously, sampled on an annual frequency and cover the period from 1930 to 2011. The "model" panel presents the predictability evidence implied by the model, based on a very long path of simulated data and the benchmark calibration.

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