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Global effective lower bound and unconventional monetary policy☆

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A B S T R A C T

In a standard open-economy New Keynesian model, the effective lower bound causes anomalies: output and terms of trade respond to a supply shock in the opposite direction compared to normal times. We introduce a tractable framework to accommodate for unconventional monetary policy. In our model, these anomalies disappear. We allow unconventional policy to be partially active and asymmetric between countries. Empirically, we find the US, Euro area, and UK have implemented a considerable amount of unconventional monetary policy: the US follows the historical Taylor rule, whereas the others have done less compared to normal times.

1. Introduction

Since the Great Recession, many major central banks of developed economies have faced the effective lower bound (ELB) for their policy interest rates and resorted to unconventional monetary policy to provide further stimulus. In this extraordinary environment, how do we evaluate the role of unconventional monetary policy theoretically and empirically?

In a standard New Keynesian model (e.g., Eggertsson and Woodford (2003) for a closed economy and Cook and Devereux (2013a) for an open economy), the ELB yields to the classic liquidity trap. The central bank cannot further reduce the policy rate, and monetary policy is completely absent. However, emerging empirical studies provide overwhelming evidence to demonstrate the effectiveness of unconventional monetary policy; see, for example, Gagnon et al. (2011), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), Bauer and Rudebusch (2014), and Wu and Xia (2016) for its domestic impact, and Neely (2015), Bauer and Neely (2014), Bowman et al. (2015), and Chen et al. (2016) for its global effects.

We propose a tractable New Keynesian model that incorporates unconventional monetary policy into an otherwise standard model to be consistent with empirical findings. We propose a Taylor (1993)-type policy rule to conveniently summarize both conventional and unconventional monetary policy; see Wu and Zhang (2017) for how to implement a negative interest rate via QE, for example. We extend the framework of Wu and Zhang (2017), where unconventional monetary policy follows the historical Taylor rule by construction. In this paper, we relax this assumption and allow unconventional policy to be potentially less effective, and countries can implement them asymmetrically. Our new model nests the traditional model where monetary policy is absent at the ELB and the model in Wu and Zhang (2017) with fully active unconventional monetary policy. We illustrate our new framework with a two-country setup, similar to Clarida et al. (2002) and Cook and Devereux (2013a), but it can be easily extended to the small-open economy.

During normal times, a negative supply shock from the home country leads to lower home output and terms of trade. In our model, if a sufficient amount of unconventional monetary policy is implemented, the same results apply for the ELB. On the contrary, the standard model implies an opposite movement of output and terms of trade during a liquidity trap, and we will refer to these movements as anomalies.

The basic mechanism that leads to these anomalies consists of two channels. First, it transmits through inflation and the real interest rate, which works the same way as in a closed-economy macro model. A negative supply shock leads to higher inflation for home goods. At the ELB, the nominal rate does not move, which lowers the real rate. The lower

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real rate stimulates demand and hence the equilibrium output of the home country. In the open economy with complete financial markets, international trade further amplifies this effect through a depreciation in terms of trade.

When we allow the two countries to implement their respective unconventional monetary policy asymmetrically, we find different results for the home and foreign economies. For the home country, its own policy matters the most, whereas the foreign economy relies on both central banks. More active home or foreign policy is associated with higher welfare, and the most efficient case is obtained when both countries' unconventional policies follow their historical policy rules.

We explore alternative model and parameter specifications for robustness. The anomalies are generally robust for alternative models with one exception: the anomaly for terms of trade depends on whether the international financial markets are complete or not, whereas the result for output is not sensitive. We also assess the robustness of these anomalies across alternative parameter values. We find they are not sensitive to structural parameters, including the Frisch elasticity of labor supply, elasticity of intertemporal substitution, and home bias. Results vary more over parameters governing the preference shock, which creates the ELB environment. We find as long as the ELB lasts for several quarters or longer, the anomalies hold.

Finally, we seek empirical evidence for unconventional monetary policy in the United States, Euro area, and United Kingdom. First, we test model implications by comparing how output responds to a supply shock in a structural vector autoregression (VAR) between normal times and the ELB. We find that for all three countries and regions, output decreases with a negative shock to the growth rate of total-factor productivity (TFP) regardless of normal times or the ELB. This result is in contrast to the anomaly presented in the standard New Keynesian model. Our theoretical model suggests unconventional monetary policy as one potential explanation for this result.[1]

Next, we quantify unconventional monetary policy empirically. Specifically, we compare what has been done with what should have been done according to the historical Taylor rule. We find the US, Euro area, and UK have implemented a considerable amount of unconventional monetary policy, which explains why the anomaly does not appear in the data. Moreover, the US operates its unconventional monetary policy similarly to the historical Taylor rule, whereas the Euro area and UK have operated less unconventionally than what they would normally have done.

Households' Euler equation is

$\beta\mathbb{E}$

Firms keep prices constant when they cannot reoptimize. Finally, the PPI evolves according to

$$P_{Ht} = \left[(1-\zeta)\tilde{P}_{Ht}^{1-\theta} + \zeta P_{H,t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{2.20}$$

2.4. Market clearing and welfare

The goods market-clearing condition is

$$Y_t = C_{Ht} + C_{Ht}^{*}. \tag{2.21}$$

The labor market clears when

$$N_t = \int_0^1 N_t(i)di. \tag{2.22}$$

Welfare W is defined as the second-order approximation of households' lifetime utility. Adding two countries together, the world welfare is

$$W^W = W + W^{*}. \tag{2.23}$$

2.5. Monetary policy and the effective lower bound

The monetary policy follows a Taylor interest-rate rule:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + (1-\rho_s)\left[\phi_\pi \hat{\pi}_{Ht} + \phi_y x_t\right], \tag{2.24}$$

where s_t is the desired interest rate, which is the interest rate implied by the Taylor rule. Hatted variables are log deviations from the steady states $\hat{s}_t = s_t - s$ and $\hat{\pi}_{Ht} = \pi_{Ht} - \pi$, $\pi = \log(\Pi)$, and s and Π are the steady-state nominal interest rate and inflation. $x_t = y_t - y_t^n$ is the output gap, $y_t = \log(Y_t)$, and $y_t^n = \log(Y_t^n)$ is the natural level of output, or the equilibrium output under flexible prices when $\zeta = 0$; see more details in Appendix A.1. ρ_s captures the persistence of the interest-rate rule, and ϕ_π and ϕ_y are the sensitivities of the nominal interest rate to inflation and output, respectively.

2.5.1. Effective lower bound and unconventional monetary policy

During normal times, the policy rate is

$$r_t = s_t.$$

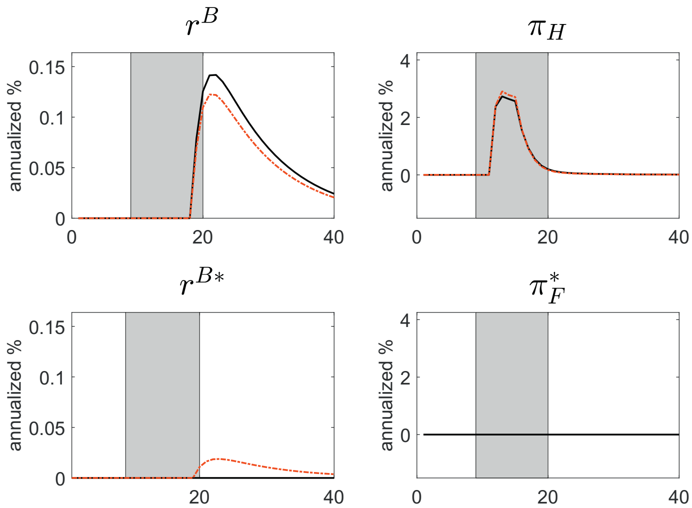
When the ELB binds $s_t < 0$,[3] the policy rate $r_t = 0$. We conveniently summarize all monetary policy actions with the shadow rate $\mathcal{S}_t$:

$$\mathcal{S}_t = \lambda s_t. \tag{2.25}$$

The case of $\lambda = 0$ and

environment with an interest-rate peg at the steady state $\hat{\mathcal{S}}_t = 0$. We find the solution that solves for any generic λ first and then impose $\lambda = 0$ for the ELB, and ignore other potential equilibria that only arise at the ELB. We will relax all these assumptions in the quantitative Section 3.1.2.

When a supply shock occurs, the in



The foreign economy, as well as welfare, relies on both central banks. A more active home unconventional policy or a less active foreign policy is associated with higher foreign inflation and output. For welfare, more active home or foreign policy is associated with higher welfare. The most efficient case happens when both countries' policies are fully active.

3.3. Alternative specifications

This section explores alternative specifications and serves as a robustness check. Section 3.3.1 explores alternative parameter spaces. Section 3.3.2 assesses an alternative monetary policy rule, Section 3.3.3 excludes trade, and Section 3.3.4 investigates incomplete financial markets.

3.3.1. Alternative parameter space

This section assesses the robustness of anomalies discussed in Subsection 3.1 across alternative parameter values, where we define anomalies when the maximum response of y and τ are positive at the ELB.[6]

Fig. 5 illustrates the existence of anomalies when we vary the persistence of the TFP dynamics ρ_a, the persistence of the preference shifter ρ_ξ, the inverse of Frisch elasticity of labor supply ϕ, elasticity of intertemporal substitution σ, home bias ν, and the length of preference shocks T_ξ, one at a time, and set other parameters as in the baseline calibration. Gray areas mark that anomalies exist, whereas white areas correspond to the parameter space where anomalies do not appear.

The anomalies are not sensitive to structural parameters ρ_a, ϕ, σ, ν: they exist as long as $\rho_a < 0.98$. This finding is consistent with the condition $0<\rho_a<\bar{\rho}_a$ in Proposition 1 that guarantees $\Lambda_a > 0$. They always exist for all $\phi \in [0.1,5]$, $\sigma \in [0.1,3]$, and $\nu \in [1,2]$.

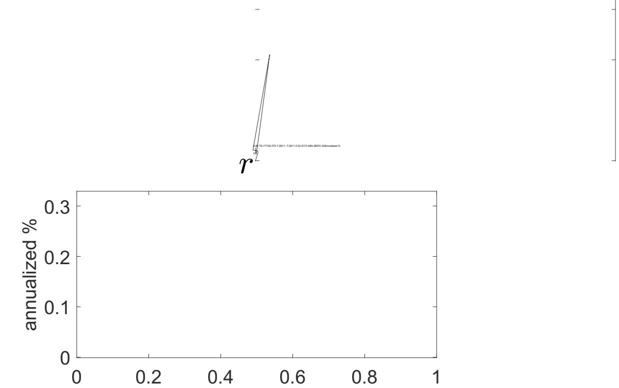
Results vary more over parameters related to the preference shock. The gray shades correspond to $0.86 \le \rho_\xi \le 0.9650$ or $T_\xi \ge 12$. Fundamentally, whether anomalies exist depends on how long the ELB lasts,[7] which varies substantially over ρ_ξ and T_ξ. When ρ_ξ is too small or too large or when T_ξ is too small, the number of ELB periods is not large enough to generate anomalies. In the case of ρ_ξ (T_ξ), anomalies are supported if ELB lasts six (three) quarters or longer.

3.3.2. CPI - based Taylor rule

Our baseline specification of the Taylor rule in (2.24) relies on the PPI inflation. A viable alternative is to have the central bank respond to the CPI inflation instead:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + (1-\rho_s)\left[\phi_\pi \hat{\pi}_t + \phi_y x_t\right]. \tag{3.4}$$

Fig. 6 shows how economic quantities vary with $\lambda = \lambda^*$ when the central bank adopts the alternative Taylor rule. The economies behave similarly to those with the PPI-based rule in Fig. 3. The impulse responses for the domestic economy and the terms of trade are lower if the monetary policy is implemented based on the CPI inflation for most λ, whereas the foreign quantities are higher in this case.



3.3.3. No-trade case

Fig. 7 plots the summary responses to the TFP shock as functions of both λ and λ* for the case with no international trade, which is instrumented by $\nu = 2$. Unlike in Fig. 4, the home economic indicators only move with the home policy indicator λ. The foreign economy in the second row does not move regardless of monetary policy. Welfare, on the other hand, still depends on monetary policies of both countries.

3.3.4. Incomplete financial markets

The benchmark model in Secition 2 and our analyses thus far assume international financial markets are complete. This section examines incomplete markets. The contrast between the red dashed line and blue solid line in the left panel of Fig. 8 demonstrates the anomaly discussed in Subsection 3.1 still exists for output. See details of the model in Appendix D.

However, whether the financial markets are complete or not does affect trade-related quantities. When the market is complete, terms of trade decrease normally in response to a negative home TFP shock. However, they increase in the setting of incomplete financial markets, which is consistent with what Enders and Müller (2009) find. Moreover, international trade lowers how much output increases at the ELB, mitigating the anomaly.

4. Empirical evidence on unconventional monetary policy

This section empirically investigates unconventional monetary policies at the ELB in the US, Euro area, and UK, and compares them with their corresponding conventional policies. First, we test model implications by comparing impulse responses in a VAR between normal times and the ELB. This exercise allows us to assess whether the anomaly exists in the data. Next, to quantify unconventional monetary policy, we rely on the Taylor rule to compare what has been done with what should have been done.

4.1. Vector autoregression

This section analyzes unconventional monetary policy in a VAR framework. We quantify empirically how output responds to a TFP shock in the US, Euro area, and UK. Then we compare our empirical results with implications from our theoretical model in Section 3 to draw conclusions.

Following Galí and Gambetti (2009), we measure TFP with labor productivity. Our VAR has two variables: the growth rate of labor productivity,[8] $\Delta(y_t - n_t)$, and the log of per-capita hours, n_t. We use a first-order VAR due to the short sample in the quarterly frequency. We identify TFP shocks through the Cholesky decomposition by ordering labor productivity first, which assumes a shock to hours has no contemporaneous impact on labor productivity growth.

We estimate the VAR for the pre-ELB and ELB samples separately. The two samples span from 1985Q2 - 2007Q4[9] and 2009Q1 - 2015Q4 for the US, 1999Q1 - 2011Q3 and 20011Q4 - 2017Q4 for the Euro area, 1993Q1 - 2009Q1 and 2009Q2 - 2017Q4 for the UK. The detailed data sources for the three countries and regions are in Appendix E.

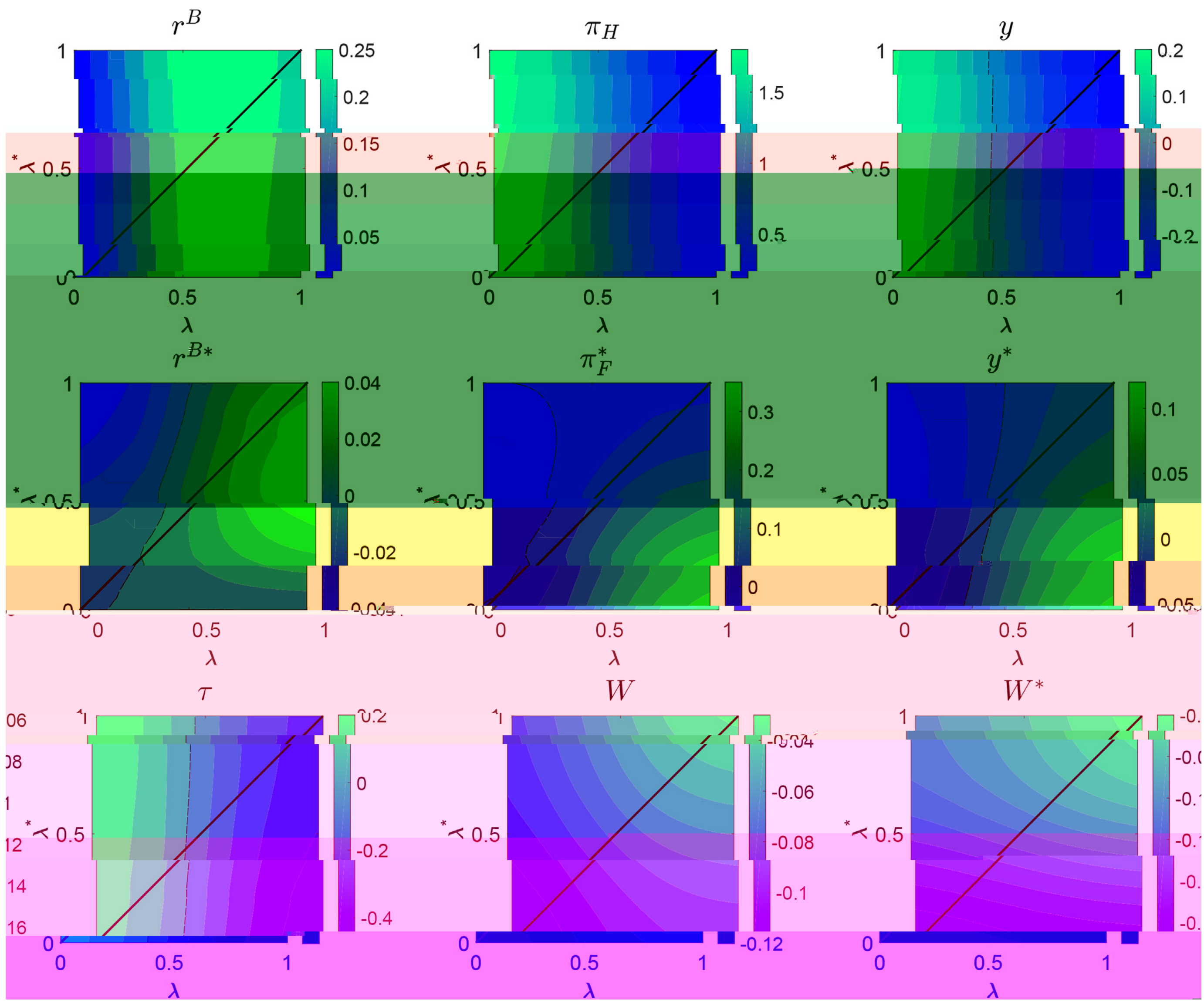


Fig. 4. Asymmetric unconventional monetary policy. Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1–15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: λ; Y-axis: λ^*. The color from light green to dark blue corresponds to high to low values. The units are annualized percentage for interest rates and inflation, percentage for output and terms of trade, and level for welfare. The 45-degree lines represent the symmetric case $\lambda = \lambda^*$. The dashed lines are the 0 contours.

Fig. 9 plots impulse responses of output to a -1% shock to labor productivity growth for the three countries and regions.[10] Blue solid lines represent normal times with medians in the thick lines, and 90% confidence intervals in the thin lines. Red dashed lines represent the central tendencies at the ELB. We find that for all three countries and regions, output decreases with a negative TFP shock regardless of normal times or the ELB. This similarity result is in contrast to the anomaly presented by the standard New Keynesian model in Subsection 3.1, and is potential evidence for unconventional monetary policy.

The left panel is for the US. We find the impulse response at the ELB is initially slightly lower than normal times, and then the red dashed and blue thick solid lines track each other closely after five quarters. Moreover, the red dashed line is within the confidence interval in blue. In the case of the US, we do not find an anomaly, and our result is consistent with Garín et al. (forthcoming) and Debortoli et al. (2016).[11]

The middle panel is for the Euro area, and the right panel is for the UK. Both of them show that output decreases less at the ELB than in normal times. The differences between normal times and the ELB are statistically significant in both cases, with the UK being more pronounced.

These findings suggest the anomaly does not appear in the data for the three countries and regions we examined. If unconventional monetary policy were the sole source that drives the difference between the standard New Keynesian model and what we see in the data, we would conclude that unconventional monetary policy is as active as usual in the US and is less active for the Euro area and UK, or $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$.

4.2. Taylor rule

In Subsection 4.1, the VAR qualitatively sorts the effectiveness of unconventional monetary policy among the three regions and countries based on our theoretical model. In this section, we quantify the amount of unconventional monetary policy implemented in each country or

[10] Output is calculated as $y_{t+j} = \sum_{\tau=0}^{j} \Delta(y_{t+\tau} - n_{t+\tau}) + n_{t+j}$, where $y_{t-1} - n_{t-1} = 0$.

[11] We find a similar comparison for Japan. Therefore, the anomaly does not exist in Japan either, which is consistent with Wieland's (forthcoming) findings. The details of the VAR analysis for Japan are in Appendix F.

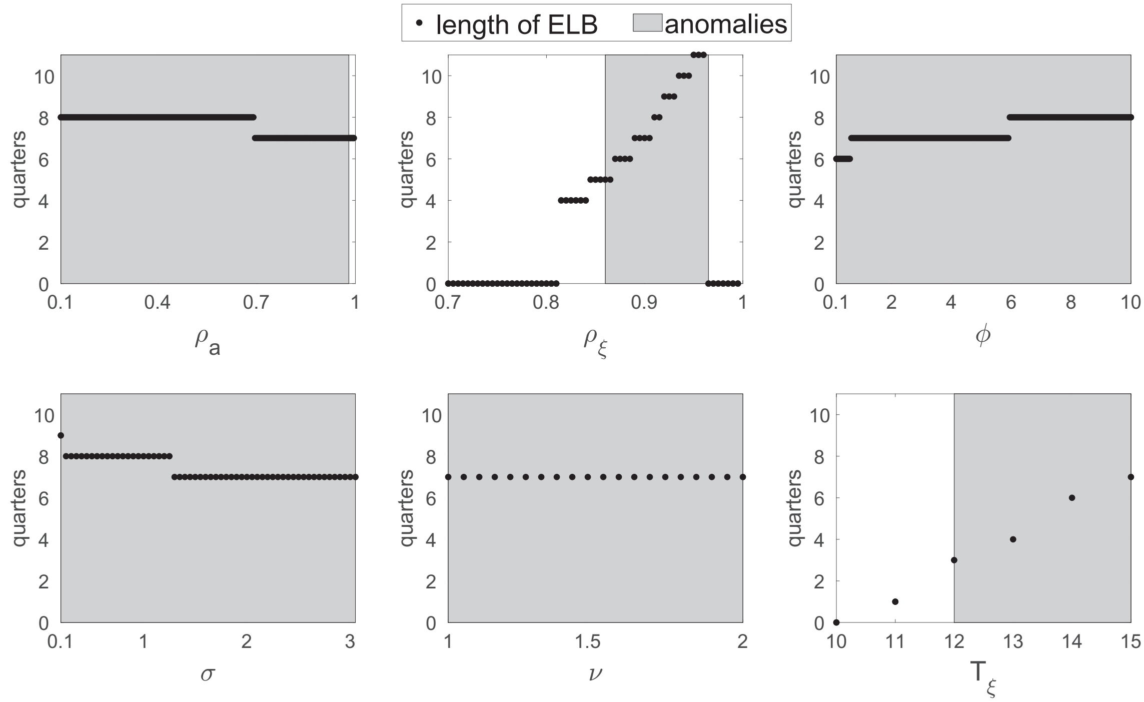


Fig. 5. Anomalies with alternative parameter values. Notes: X-axis: ρ_a in the top left panel, ρ_ξ in the top middle panel, ϕ in the top right panel, σ in the bottom left panel, ν in the bottom middle panel, and T_ξ in the bottom right panel. Y-axis: time in quarters. Black dots: the number of ELB periods after the TFP shock. Gray shades: anomalies exist; white areas: anomalies do not exist.

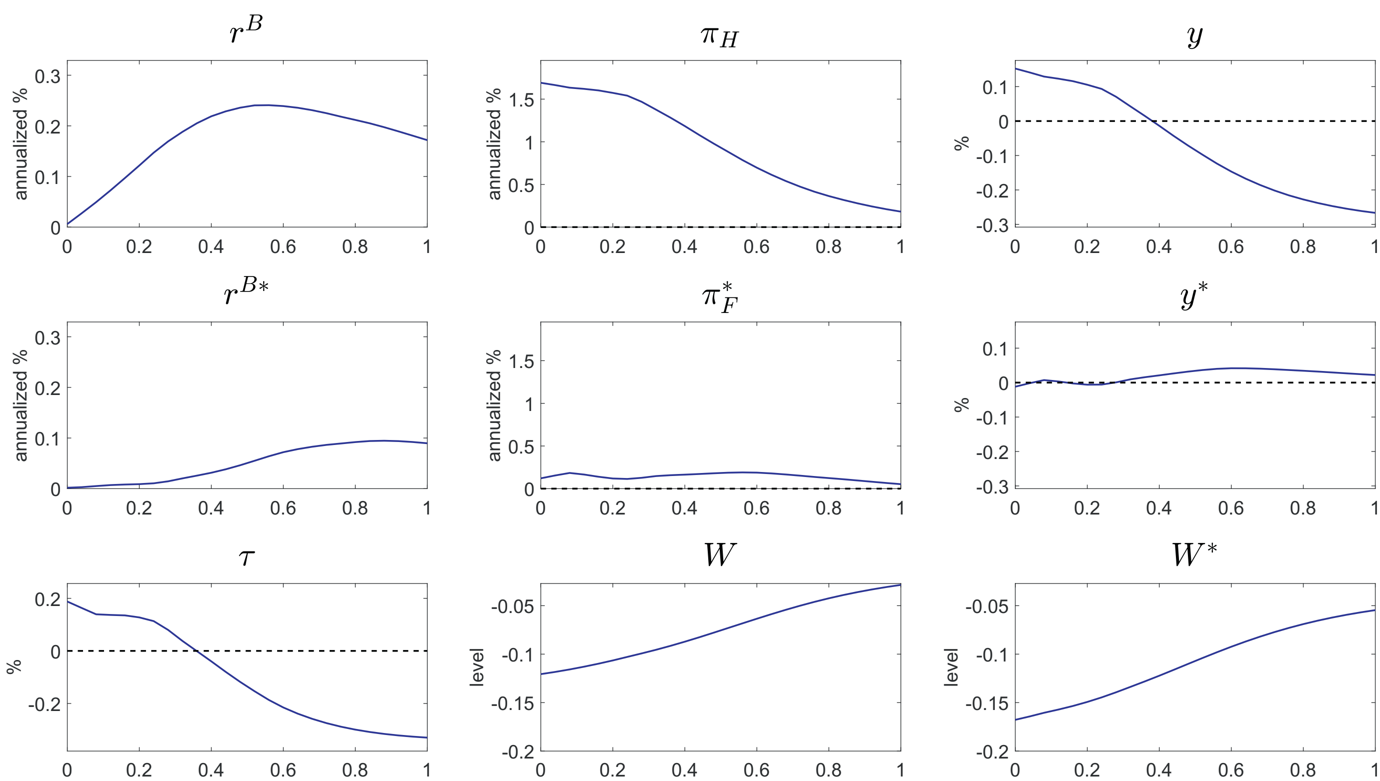


Fig. 6. CPI vs. PPI - based Taylor rule. Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1–15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: $\lambda = \lambda^*$. Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

region, and assess whether this amount we observe can explain the difference between the standard New Keynesian model and what we find in our VAR.

We quantify unconventional monetary policy by comparing what has been done at the ELB, measured by the shadow rates of Wu and Xia (2016) and Wu and Xia (2017),[12] with the desired interest rates implied by the historical Taylor rule.

We estimate the historical Taylor rule,

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 \pi_{Ht} + \beta_3 x_t + \varepsilon_t, \tag{4.1}$$

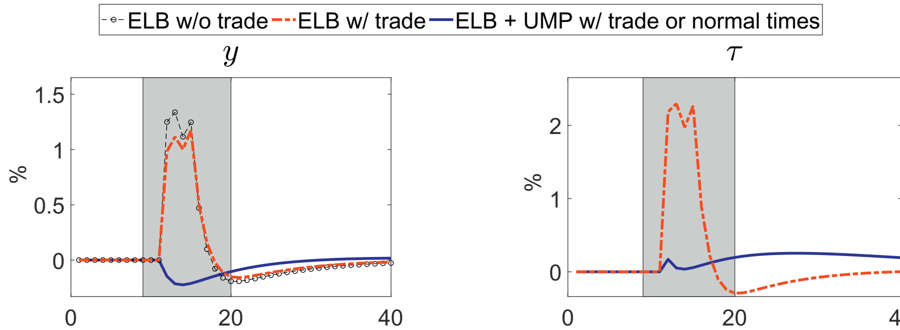


Fig. 8. Incomplete markets. Notes: The international financial markets are incomplete. A negative technological shock of -0.5% happens in the home country in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 − 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The black dashed line with circles represents the ELB case without trade, the red dashed lines represent the case with trade, and the blue solid lines represent the case with trade and UMP, which is the same as in normal times. The shaded area marks periods 9 - 20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: percentage changes relative to the steady states.

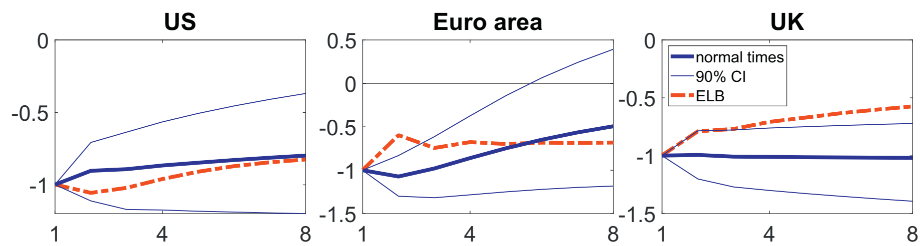


Fig. 9. Impulse response of output to a productivity shock. Notes: Impulse responses of output to a -1% shock to labor productivity growth. The blue solid lines are normal times, with thick lines being medians and thin lines representing 90% confidence intervals. The red dashed lines are the median impulse responses at the ELB. X-axis: time in quarters; Y-axis: percentage changes in output.

are the same as in Subsection 4.1. The details of the data are in Appendix E. The estimate of the simple method is 1.02, and is 1.12 from the iterative method. We conclude that the US unconventional monetary policy is as active as, if not more active than, the historical Taylor rule.

The Taylor rule is known to vary over different sample periods, and researchers' choices of sample periods in the literature are far from unanimous. We quantify the variation of our estimates by varying the pre-ELB estimation sample: the beginning of the sample ranges from $t_0 \in \{1982Q1 : 1990Q1\}$ and the end of the sample varies from $t_1 \in \{2003Q1 : 2008Q4\}$, which covers the majority of popular choices. We compute a λ for each combination of t_0 and t_1 and plot its distribution across all possible combinations in Fig. 10. The left panel plots the distribution for the simple method, and the right panel uses the iterative method. They both center around 1: the median for the simple method is 1.03, and is 1.19 for the iterative method. The standard error for the simple method is 0.065, and is 0.45 for the iterative method. The iterative method displays a larger variation across different sample periods than the simple method. On the other hand, the results from the simple method might be biased if λ is far from 1. This is the classic bias-variance trade off.

For the Euro area and UK, quarterly real potential GDP is not available. Hence, we replace x_t in (4.1) with output growth Δy_t, measured by the growth rate of real GDP. The r_t for the Euro area is the 3-month Euro Interbank Offered Rate (Euribor), and it is the Bank of England

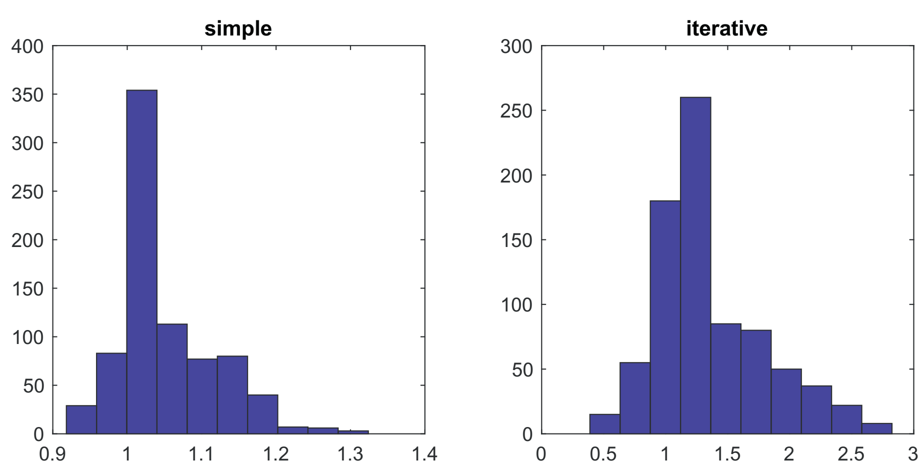


Fig. 10. Distribution of λ for the US. Notes: $t_0 \in \{1982Q1 : 1990Q1\}$, and $t_1 \in \{2003Q1 : 2008Q4\}$. For each combination of t_0 and t_1, estimate a λ from t_0 to t_1. Then plot the distribution across all possible combinations of t_0 and t_1. Left panel: simple method; right panel: iterative method.

policy rate for the UK. The details of the data are in Appendix E. For the Euro area, $t_0 \in \{1998Q2 : 1999Q1\}$ and $t_1 \in \{2009Q1 : 2011Q3\}$. The ELB period is from t_1

where $\Theta = \frac{(1-\beta\zeta)(1-\zeta)}{\zeta} > 0$. The definitions for terms of trade (2.12) and nominal exchange rate (2.14) are

$$\hat{\tau}_t = p_{Ft} - p_{Ht} \tag{A.25}$$

$$p_{Ht} = e_t + p^*_{Ht} \tag{A.26}$$

$$p_{Ft} = e_t + p^*_{Ft}. \tag{A.27}$$

Combining (A.19) – (A.22) and (A.25) – (A.27), the CPI inflation can be expressed as a function of PPI inflation and terms of trade:

$$\hat{\pi}_t = \hat{\pi}_{Ht} + \left(1 - \frac{\nu}{2}\right)\Delta\hat{\tau}_t \tag{A.28}$$

$$\hat{\pi}^*_t = \hat{\pi}^*_{Ft} - \left(1 - \frac{\nu}{2}\right)\Delta\hat{\tau}_t. \tag{A.29}$$

The labor-supply decision (A.2) in the flexible-price economy becomes

$$\sigma\hat{c}^n_t + \phi\hat{n}^n_t = \hat{a}_t + (\nu/2 - 1)\hat{\tau}^n_t \tag{A.30}$$

$$\sigma\hat{c}^{n*}_t + \phi\hat{n}^{n*}_t = \hat{a}^*_t - (\nu/2 - 1)\hat{\tau}^n_t. \tag{A.31}$$

The international risk-sharing condition (A.3) in the flexible-price economy is

$$\sigma\left(\hat{c}^n_t - \hat{c}^{n*}_t\right) - \left(\hat{\xi}_t - \hat{\xi}^*_t\right) - (\nu - 1)\hat{\tau}^n_t = 0. \tag{A.32}$$

The market-clearing conditions (A.4) and (A.5) in the flexible-price economy are

$$\hat{y}^n_t = \left[\frac{\nu}{2}\hat{c}^n_t + \left(1 - \frac{\nu}{2}\right)\hat{c}^{n*}_t\right] + \nu\left(1 - \frac{\nu}{2}\right)\hat{\tau}^n_t \tag{A.33}$$

$$\hat{y}^{n*}_t = \left[\frac{\nu}{2}\hat{c}^{n*}_t + \left(1 - \frac{\nu}{2}\right)\hat{c}^n_t\right] - \nu\left(1 - \frac{\nu}{2}\right)\hat{\tau}^n_t. \tag{A.34}$$

The output gaps are defined as

$$x_t = y_t - y^n_t \tag{A.35}$$

$$x^*_t = y^*_t - y^{n*}_t. \tag{A.36}$$

Eqs. (A.6) to (A.36) and the monetary policy rules (2.24) and (2.25) where $K = \sigma + \phi - K_1$. The foreign country's counterpart is

$$\widehat{mc}_t^* = K\hat{y}_t^* + K_1\hat{y}_t. \tag{B.13}$$

Combining (B.12) and (B.13) with (A.23) and (A.24), the New Keynesian Phillips curves (NKPCs) are

$$\hat{\pi}_{Ht} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \Theta K\hat{y}_t - \Theta(1+\phi)\hat{a}_t + \Theta K_1\hat{y}_t^* \tag{B.14}$$

$$\hat{\pi}_{Ft}^* = \beta\mathbb{E}_t\hat{\pi}_{F,t+1}^* + \Theta K\hat{y}_t^* + \Theta K_1\hat{y}_t. \tag{B.15}$$

The difference is

$$\begin{aligned}\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^* = \beta\mathbb{E}_t\left(\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*\right) + \Theta(K - K_1)\\ \times(\hat{y}_t - \hat{y}_t^*) - \Theta(1+\phi)\hat{a}_t.\end{aligned} \tag{B.16}$$

Next, we solve the system of equations in (B.11) and (B.16). When $\lambda\phi_\pi > 1$, the Blanchard-Kahn condition is satisfied, and the system has a unique solution, which is (3.1), (3.2). Next, (B.1) implies (3.3).

In our model, $\Theta > 0$, $1 + \phi > 0$, $1 - \rho_a > 0$, $D > 0$, $D + 1 > 0$, $\sigma > 0$, $\sigma_0 > 0$.

- When $\lambda = 1$ and $\phi_\pi > 1$, $\Lambda_a > 0$ and $\lambda\phi_\pi - \rho_a > 0$.
- When $\lambda = 0$, the denominator of Λ_a is a convex quadratic function of ρ_a with one root between 0 and 1 and another root larger than 1. We solve the root within the unit circle

$\overline{\rho}_a = \frac{2\sigma_0(1+\beta)+\Theta(\sigma/D+\phi)(D+1)-\sqrt{[2\sigma_0(1+\beta)+\Theta(\sigma/D+\phi)(D+1)]^2-16\sigma_0^2\beta}}{4\sigma_0\beta}$, and $0 < \rho_a < \overline{\rho}_a$ guarantees $\Lambda_a > 0$. Moreover, $\lambda\phi_\pi - \rho_a < 0$.

B.2. Proof of Corollary 1

When $\sigma = 1$ or $\nu = 2$, $K_1 = K_2 = 0$, so that $\sigma_0 = \sigma$, $K = \sigma + \phi$, and $D = 1$. For the foreign economy, (B.10) and (B.15) yield to

$$\hat{\pi}_{Ft}^* = \hat{y}_t^* = 0. \tag{B.17}$$

The solution to (B.9) and (B.14) for the home economy is

$$\hat{y}_t = \Theta(\lambda\phi_\pi - \rho_a)(1+\phi)\Lambda_a\hat{a}_t \tag{B.18}$$

$$\hat{\pi}_{Ht} = -\Theta(1-\rho_a)(1+\phi)\Lambda_a\hat{a}_t, \tag{B.19}$$

and (B.1) implies $\hat{\tau}_t = \sigma\hat{y}_t$.

B.3. Proof of Proposition 2

When $\lambda = 0$, (3.2) and (3.3) become

$$\hat{y}_t - \hat{y}_t^* = -\rho_a\Theta(1+\phi)(D+1)\Lambda_a\hat{a}_t \tag{B.20}$$

$$\hat{\tau}_t = -\rho_a\Theta(1+\phi)\frac{\sigma(D+1)}{D}\Lambda_a\hat{a}_t. \tag{B.21}$$

First,

$$\frac{\partial D}{\partial \nu} = 2(1-\sigma)(\nu-1) < 0,$$

given $\sigma > .5$(h)0(o/F41Tf-29.4.6TD(D)Tj/F13 1

Appendix D. Incomplete asset markets

Following Benigno (2009), there is no longer complete international risk sharing in the model with incomplete asset markets, where only two nominal non-contingent bonds are traded. Then the international risk sharing condition (2.16) no longer holds, and the household's budget constraint (2.2) in the baseline model changes to:

$$P_tC_t - \frac{B_{Ht+1}}{R_t^B} + \frac{\varepsilon_t B_{Ft+1}}{R_t^{B*}} + \frac{\ell P_t}{2R_t^{B*}}\left(\frac{\varepsilon_t B_{Ft+1}}{P_t} - \bar{\iota}\right)^2 = \varepsilon_t B_{Ft} - B_{Ht} + W_tN_t + TR_t + \mathcal{D}_t, \tag{D.1}$$

where B_{Ht+1} is the debt issued in units of risk-free nominal bond denominated in H currency, and the nominal interest rate on this bond is R_t^B. B_{Ft+1} is the holding of risk-free nominal bond denominated in units of foreign currency, and the nominal interest rate on this bond is R_t^{B*}. The assumption that households of the home country hold assets denominated in foreign currency and issue debt in the home currency, reflects the net international position of the US economy. We assume a quadratic transition cost when deviating the real foreign bond position from a constant real value, denoted by $\bar{\iota}$; ℓ is nonnegative, measuring this cost in terms of units of the consumption index, and is rescaled by the factor $1/R_t^{B*}$ for analytical convenience. The quadratic cost serves for the purpose of determining the steady state and getting rid of the indeterminacy problem. TR_t includes government transfer and the revenues obtained from the transaction costs paid by households in the foreign country when trading home country bonds, and $\mathcal{D}_t$ is the profits from firms.

The first-order conditions of home country households with respect to domestic and foreign bonds are:

$$\beta \mathbb{E}_t\left[\frac{\Xi_{t+1}}{\Xi_t}\frac{C_t^{\sigma}}{C_{t+1}^{\sigma}}\frac{R_t^B}{\Pi_{t+1}}\right] = 1 \tag{D.2}$$

$$\beta \mathbb{E}_t\left[\frac{\Xi_{t+1}}{\Xi_t}\frac{C_t^{\sigma}}{C_{t+1}^{\sigma}}\frac{R_t^{B*}}{\Pi_{t+1}}\frac{\varepsilon_{t+1}}{\varepsilon_t}\right] = 1 + \ell\left(\frac{\varepsilon_t B_{Ft+1}}{P_t} - \bar{\iota}\right). \tag{D.3}$$

Utility maximization of households in the foreign country yields the counterparts of Eqs. (D.2) and (D.3). The equilibrium in the asset markets requires that

$$B_{Ht} - B_{Ht}^* = 0 \tag{D.4}$$

$$B_{Ft} - B_{Ft}^* = 0. \tag{D.5}$$

Combining the household's and government budget constraints, we obtain the aggregate budget constraint of the home country:

$$P_tC_t - \frac{B_{Ht+1}}{R_t^B} + \frac{\varepsilon_t B_{Ft+1}}{R_t^{B*}} + \frac{\ell P_t}{2R_t^{B*}}\left(\frac{\varepsilon_t B_{Ft+1}}{P_t} - \bar{\iota}\right)^2 = \varepsilon_t B_{Ft} - B_{Ht} + P_{Ht}Y_t + \frac{\ell^* P_t}{2R_t^B}\left(\frac{B_{Ht+1}^*}{\varepsilon_t P_t^*} - \bar{\iota}^*\right)^2. \tag{D.6}$$

Appendix E. Data

- Shadow rates are downloaded from Cynthia Wu's website:

https://sites.google.com/site/jingcynthiawu/home/wu-xia-shadow-rates.

- The U.S. macroeconomic variables are downloaded from the Database of the Federal Reserve Bank of St. Louis (FRED) at http://research.stlouisfed.org/fred2/.
  - Real GDP (GDPC): billions of chained 2009 dollars, seasonally adjusted.
  - Real potential GDP (GDPPOT): billions of chained 2009 dollars, not seasonally adjusted.
  - GDP deflator (GDPDEF): index 2009 = 100, seasonally adjusted.
  - Effective federal funds rate (FEDFUNDS): percent.
  - Real output per hour of all persons (nonfarm business sector) (OPHNFB): index 2009 = 100, seasonally adjusted.
  - Hours of all persons (nonfarm business sector) (HOANBS): index 2009 = 100, seasonally adjusted.
  - Civilian noninstitutional population (CNP16OV): thousands of persons.
- The Euro area macroeconomic variables are from the ECB Statistical Data Warehouse at http://sdw.ecb.europa.eu/home.do
  -

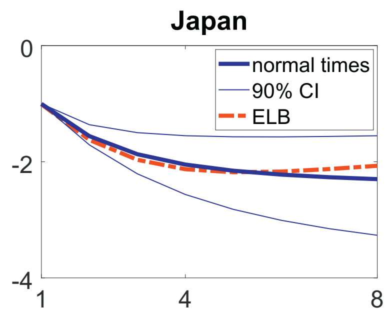


Fig. F1. Impulse response of output to a productivity shock. Notes: Impulse responses of output to a -1% shock to labor productivity growth. The blue solid lines are normal times, with thick lines being the median and thin lines representing 90% confidence intervals. The red dashed line is the median impulse response at the ELB. X-axis: time in quarters; Y-axis: percentage changes in output.

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