



PBC School of Finance, Tsinghua University, China

ARTICLE INFO

ABSTRACT



1. Introduction



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:// . /10.1016/jj .2019.103728 0165-1889/ 2019 .V.



¹ (2012), . (2014), . (2016), . (2017). уу у _ (2014) (2014), (2014), у (2014), (2016), _J _ , (2016). (2015),



2. A shadow rate New Keynesian model (SRNKM)



2.1. Standard NK model and its lack of unconventional monetary policy









2.5. Economic implications



$$C_t + \frac{B_t^H}{P_t} = \frac{R_{t-1}^B B_{t-1}^H}{P_t} + W_t L_t + T_t,$$
(3.2)

$$R^B_{t-1}$$
. P_t , W_t , T_t - . . - $B^H_t \equiv B^H_t / P_t$

$$C_t^{-\sigma} = \beta R_t^{\mathcal{B}} \mathbb{E}_t \left(\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \right), \tag{3.3}$$
$$\Pi_{t+1} \equiv P_{t+1}/P_t \qquad \qquad t \quad t+1.$$
$$- \qquad \qquad Y_t = C_t \ \mathbf{y} \qquad \qquad :$$

$$y_{t} = -\frac{1}{\sigma} \left(r_{t}^{B} - \mathbb{E}_{t} \pi_{t+1} - r^{B} \right) + \mathbb{E}_{t} y_{t+1}.$$
(3.4)

(2.1)

i

interest

 B_{t-1}^H

$$rp_{t} \equiv r_{t}^{B} - r_{t},$$

$$y \quad r_{t} \qquad y \qquad (2.3) \quad (2.4) \qquad (3.5)$$

$$y \quad .$$
the term
$$y \quad .$$

$$(3.4) (3.5) (3.7), (2.2) y (2.3) (2.4).$$

3.2. Shadow rate equivalence for QE

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y y (3.4)

$$r_t^B = r_t + rp - \zeta (b_t^{CB} - b^{CB}).$$
 (3.8)
, $b_t^{CB} = b^{CB}, r_t^B = r_t + rp,$ y y y $r_t,$ (3.8)

$$r_t^B = rp - \varsigma (b_t^{CB} - b^{CB}).$$

$$b_t^{CB} = b^{CB} - \frac{s_t}{z},$$
(3.9)

$$r_{t}^{B} = s_{t} + rp$$
(3.10)
$$y \quad y \quad y \quad y \quad .$$
(3.8)
$$y \quad r_{t} \quad - y \quad .$$
(3.10)

Proposition 1. The shadow rate New Keynesian model represented by the shadow rate IS curve (2.6), New Keynesian Phillips curve (2.2), and shadow rate Taylor rule (2.3) is equivalent to a model where monetary policy is implemented by the conventional Taylor rule during normal times and QE at the ZLB that changes the risk premium through (3.7) if

$$\begin{cases} r_t = s_t, \ b_t^{CB} = b^{CB} & s_t \geq 0 \\ r_t = 0, \ b_t^{CB} & (3.9) & s_t < 0. \end{cases}$$

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Proof.



3.3. Quantifying the assumption in Proposition 1



3.4. Announcement effect of QE

		(2012)	. (2011)			
•	(2012)				•	. (2011),
,	, y (3.7)	:		•		





is equivalent to the model summarized by (2.3)-(2.4) and (4.12)-(4.19), where monetary policy is implemented by the conventional Taylor rule during normal times and lending facility – tax policy at the ZLB if

$$\begin{cases} r_t = s_t, \ \tau_t = 0, \ m_t = m & s_t \ge 0 \\ r_t = 0, \ \tau_t = m_t - m = -s_t & s_t < 0. \end{cases}$$

. 🗆

Proof.

у

5. Partially active monetary policy

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У	, ,		У		Y	у –
	у.	у у	У	?		
	у,					

у

Definition 2.

$$y_t = -\frac{1}{\sigma} (\mathcal{S}_t - \mathbb{E}_t \pi_{t+1} - \mathcal{S}) + \mathbb{E}_t y_{t+1},$$
(5.1)

y (2.2), y (2.3),

$$\begin{cases} S_t = s_t & s_t \ge 0\\ S_t = \lambda s_t & s_t < 0, \end{cases}$$
(5.2)

$$0 \leq \lambda \leq 1$$
 y y .

у

(5.2)
$$S_t = \left(\frac{exp(\varphi s_t)}{1 + exp(\varphi s_t)}(1 - \lambda) + \lambda\right) s_t,$$
(5.3)

6. Quantitative analyses

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í) 2.5.	•	У	-	. ,	

6.1. Model and methodology







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nominal rate Ā . ١





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$$\begin{split} & I_{t} & \ddots & P_{t}^{L}, \\ & X_{t} \equiv P_{t}/P_{t}^{E}, \\ & & C_{t}^{E}, \\ & & I_{t}, \\ & L_{t} & y \\ & \mathbb{E}_{0}\sum_{t=0}^{\infty}\gamma^{t} & C_{t}^{E}, \\ & & & & I_{t}, \\ & & L_{t} & y \\ & \mathbb{E}_{0}\sum_{t=0}^{\infty}\gamma^{t} & C_{t}^{E}, \\ & & & & I_{t}, \\ & & & I_$$

$$c_{t} = -\frac{1}{\sigma} (r_{t}^{B} - \mathbb{E}_{t} \pi_{t+1} - r^{B}) + \mathbb{E}_{t} c_{t+1}, \qquad (.7)$$

$$C^{E}c_{t}^{E} = \alpha \frac{Y}{X}(y_{t} - x_{t}) + Bb_{t} - R^{B}B(r_{t-1}^{B} + b_{t-1} - \pi_{t-1}) - Ii_{t} + \Lambda_{1}, \qquad (.8)$$

$$b_t = \mathbb{E}_t (k_t + \pi_{t+1} + m - r_t^B),$$
(.9)

$$0 = \left(1 - \frac{M}{R^{B}}\right)(c_{t}^{E} - \mathbb{E}_{t}c_{t+1}^{E}) + \frac{\gamma\alpha Y}{XK}\mathbb{E}_{t}(y_{t+1} - x_{t+1} - k_{t}) + \frac{M}{R^{B}}\mathbb{E}_{t}(\pi_{t+1} - r_{t}^{B}) + \Lambda_{2},$$
(10)

$$y_t = \frac{\alpha(1+\eta)}{\alpha+\eta} k_{t-1} - \frac{1-\alpha}{\alpha+\eta} (x_t + \sigma c_t) + \frac{1+\eta}{\alpha+\eta} a + \frac{1-\alpha}{\alpha+\eta} \qquad (1-\alpha),$$
(11)

$$k_t = (1 - \delta)k_{t-1} + \delta i_t - \delta \qquad \delta, \tag{12}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} - \lambda (x_t - x), \tag{13}$$

$$y_t = \frac{C}{Y}c_t + \frac{C^E}{Y}c_t^E + \left(1 - \frac{C}{Y} - \frac{C^E}{Y}\right)i_t, \qquad (.14)$$

 $\Lambda_{1} = C^{E} \quad C^{E} - \alpha \frac{\gamma}{X} \quad \frac{\gamma}{X} - B \quad B + R^{B}B \quad R^{B}B + I \quad I, \ \Lambda_{2} = -\frac{\gamma \alpha Y}{XK} \quad \frac{\gamma}{XK} + \frac{M}{R^{B}} \quad R^{B}. \quad \Lambda_{2} \quad (4.15) \quad \Lambda_{2} = \Lambda_{2} - \left(\frac{1}{R^{B}} - \gamma\right)M \quad M.$ Equivalence. , 1

Corollary 1. The shadow rate New Keynesian model represented by the shadow rate IS curve

$$c_t = -\frac{1}{\sigma}(s_t - \mathbb{E}_t \pi_{t+1} - s) + \mathbb{E}_t c_{t+1},$$
(.15)

the shadow rate Taylor rule (2.3), together with (A.11)-(A.14) and

$$C^{E}c_{t}^{E} = \alpha \frac{Y}{X}(y_{t} - x_{t}) + Bb_{t} - R^{B}B(s_{t-1} + rp + b_{t-1} - \pi_{t-1}) - Ii_{t} + \Lambda_{1}, \qquad (.16)$$

$$b_t = \mathbb{E}_t (k_t + \pi_{t+1} + m - s_t - rp), \qquad (17)$$

$$0 = \left(1 - \frac{M}{R^{B}}\right)(c_{t}^{E} - \mathbb{E}_{t}c_{t+1}^{E}) + \frac{\gamma\alpha Y}{XK}\mathbb{E}_{t}(y_{t+1} - x_{t+1} - k_{t}) + \frac{M}{R^{B}}\mathbb{E}_{t}(\pi_{t+1} - s_{t} - rp + m) + \Lambda_{2}$$
(18)

is equivalent to the model summarized by (2.3), (2.4), (3.5), (3.7), and (A.7)–(A

(5.3):

Proof for Proposition 4. -

$$f \text{ for Proposition 4.} - \frac{exp(\varphi s_t)}{1+exp(\varphi s_t)} \quad (5.3):$$

$$\frac{exp(\varphi s_t)}{1+exp(\varphi s_t)} = \frac{exp(\varphi s)}{1+exp(\varphi s)} + \frac{exp(\varphi s)}{1+exp(\varphi s)}\varphi s_t - \frac{exp(2\varphi s)}{(1+exp(\varphi s))^2}\varphi s_t, \quad (.3)$$

$$s_t = s_t - s_t \quad (5.3)$$

$$\frac{exp(\varphi s)}{1 + exp(\varphi s)} = \frac{\frac{s}{s} - \lambda}{1 - \lambda}$$
(.4)

$$S/s = \omega + (1 - \omega)\lambda.$$
(.5)

$$y \qquad (.4), \ \frac{exp(\varphi s)}{1+exp(\varphi s)} = \omega. \qquad y \qquad (.3) \qquad (5.3).$$
$$\mathcal{S}_t = \left(\left[1 + \varphi(1-\omega)s_t \right] \omega(1-\lambda) + \lambda \right) s_t. \qquad (.6)$$

$$S_{t} + S = \left(\left[1 + \varphi(1 - \omega)s_{t} \right] \omega(1 - \lambda) + \lambda \right) (s_{t} + s) \\ = \left(\left[1 + \varphi(1 - \omega)s_{t} \right] \omega(1 - \lambda) + \lambda \right) s_{t} + \left(\left[1 + \varphi(1 - \omega)s_{t} \right] \omega(1 - \lambda) + \lambda \right) s_{t} \\ = \omega(1 - \lambda)s_{t} + \lambda s_{t} + \omega(1 - \lambda)s + \varphi s(1 - \omega)\omega(1 - \lambda)s_{t} + \lambda s \\ y \qquad (.5), \\ S_{t} = \omega(1 - \lambda) + \lambda + \varphi s(1 - \omega)\omega(1 - \lambda) s_{t}, \\ (5.4). \square$$
For Proposition 5. (5.6), (5.4)

Proof for Proposition 5.

(.8) $S_t = s_t$.

Appendix C. Quantitative model

C.1. Setup

C.1.1. Patient households

C.1.2. Impatient households

$$\frac{Q_t}{C_t^l} = \frac{j}{H_t^l} + \mathbb{E}_t \left[\beta^l \frac{Q_{t+1}}{C_{t+1}^l} \left(1 - \frac{\widetilde{M}_t M_t^l}{\mathcal{T}_t} \right) + \frac{\widetilde{M}_t M_t^l Q_{t+1} \Pi_{t+1}}{C_t^l R_t^B} \right].$$

$$(.8)$$

C.1.3. Entrepreneurs

Y

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$$y - i = T_{t}^{G} + \mathcal{T}_{t}^{P} + \mathcal{T}_{t}^{I}, \qquad (.22)$$

$$G_{t} - i = \int_{G}^{G_{t}} \int_{G_{t}}^{\rho_{g}} \varepsilon_{g,t}, \qquad (.23)$$

$$\varepsilon_{g,t} - i = \int_{Y}^{G_{t}} \mathcal{T}_{t}^{P}(\mathcal{T}_{t}^{I}) - i = (...)$$

$$\mathcal{T}_{t}^{P} = \alpha(G_{t} - T_{t}^{G}) \qquad (.24)$$

$$\mathcal{T}_{t}^{I} = (1 - \alpha)(G_{t} - T_{t}^{G}).$$
(.25)

C.1.6. Equilibrium

$$\{ H_t^E, H_t^P, H_t^I, L_t^E, L_t^P, L_t^I, Y_t, C_t^E, C_t^P, C_t^I, I_t, K_t, B_t^E, B_t^P, B_t^I, B_t^{CB}, G_t, \Delta \mathcal{M}_t^{CB}, \Delta \mathcal{M}_t^P, \Delta \mathcal{M}_t^I, \\ \{ W_t^P, W_t^I, S_t, P_t, P_t^*, X_t, Q_t \}_{t=0}^{\infty}, \\ - : H_t^E + H_t^P + H_t^I = H, C_t^E + C_t^P + C_t^I + I_t + G_t = Y_t, B_t^P + B_t^{CB} = B_t^E + B_t^I, (.20), (.22).$$

C.2. Calibration

.1			. у		(2	2005),	-V	. (2015),
у-	(2019)	Λ 	(2007). 1.005,	2%	Ϋ́		у-	
2003 2007,	2%	б. у-	У		0.9%	1	у 3.6%	

Table C.1

			v
β		(2005)	0.99
β^{I}		(2005)	0.95
Y		(2005)	0.98
j	у-	(2005)	0.1
η	У	(2005)	0.01
μ		(2005)	0.3
ν		(2005)	0.03
δ		(2005)	0.03
X	У	(2005)	1.05
θ	у -	(2005)	0.75
α		(2005)	0.64
ME		(2005)	0.89
M^{I}		(2005)	0.55
ϕ_s		(2005)	0.73
ϕ_y	ſ	(2005)	0.27
ϕ_{π}	· •	(2005)	0.13
Ÿ	у	-V . (2015)	0.20
$ ho_a$	У	-V . (2015)	0.90
$ ho_g$	-	-V . (2015)	0.80
$ ho_{eta}$		-V . (2015)	0.80
ρ_M	У	(2019)	0.98
ξp		2007)	0.24
II pCB	y- 1	2%	1.005
$\frac{B}{B^E + B^I}$	у-		0.02
\mathcal{T}	у- (у)	(y)	1
rp	У-	3.6% y	1.009

$$\frac{H^{P}}{H^{P}} = \frac{QH^{I}}{Y} / \frac{QH^{E}}{Y}.$$
(140)
$$(140)$$

$$\frac{H^E}{H^P} = \frac{QH^E}{Y} / \frac{QH^P}{Y}$$
(.40)

$$s^{P} = \alpha (1 - \mu - \nu) + X - 1 / X$$

-

:

$$\frac{QH^p}{Y} = \frac{QH^p}{C^p} \frac{C^p}{Y},$$
(.39)

$$\frac{QH^p}{C^p} = \frac{j}{1-\beta}$$
(.38)

$$\frac{C^{P}}{Y} = s^{P} - (1 - \alpha)\frac{G - T^{G}}{Y} + (RR^{B} - 1)\frac{B^{P}}{Y}$$
(.37)

$$\frac{B^{P}}{V} = \frac{B^{E}}{V} + \frac{B^{I}}{V} - \frac{B^{CB}}{V}$$
(.36)

-

$$T^{G} = \left(\frac{R^{B}}{\Pi} - 1\right)B^{CB}.$$
(.35)

$$s^{I} = \frac{(1-\alpha)(1-\mu-\nu)}{X} \qquad (20)$$

$$\overline{QH^{I}} = \frac{R^{B}}{R^{B}}$$

$$(.33)$$

$$\frac{C^{I}}{Y} = \frac{S^{I} - \alpha \frac{G - T^{G}}{Y}}{1 + \frac{QH^{I}}{R} + 1 + \frac{R^{I}}{R}},$$

$$(.34)$$

$$\frac{C^{l}}{C^{l}} = \frac{1}{1 - \beta^{l} (1 - M^{l}) - \frac{M^{l}}{RR^{B}}}$$
(32)
$$\frac{B^{l}}{CH^{l}} = \frac{M^{l} \Pi}{R^{B}}$$
(33)

$$\frac{QH^{I}}{C^{I}} = \frac{j}{1 - \beta^{I}(1 - M^{I}) - \frac{M^{I}}{1 - \beta^{I}}}$$
(32)

$$\frac{QH^{l}}{j} = \frac{j}{j}$$
(32)

$$\gamma^{c} = (1 - M^{c})\gamma + M^{c}p.$$

$$\overline{Y} = \left[\mu + \nu - \frac{1}{1 - \gamma(1 - \delta)} - (1 - \beta)MX \overline{Y}\right]\overline{X},$$

$$\gamma^{e} = (1 - M^{E})\gamma + M^{E}\beta.$$
(51)

$$\frac{C^{E}}{Y} = \left[\mu + \nu - \frac{\delta\gamma\mu}{1 - \nu(1 - \delta)} - (1 - \beta)M^{E}X\frac{QH^{E}}{Y}\right]\frac{1}{X},$$
(.31)

$$\frac{QH}{Y} = \frac{\gamma \nu}{X(1-\gamma^e)}$$

$$\frac{B^E}{X} = \beta M^E \frac{QH^E}{X}$$
(.29)

$$\frac{QH^{E}}{Y} = \frac{\gamma v}{X(1-\gamma^{e})}$$
(.29)

$$\frac{QH^E}{Y} = \frac{\gamma \nu}{X(1-\gamma^e)}$$
(.29)

$$QH^E = \gamma \nu$$

$$KK = 1/p.$$
 (1.20)

$$RR^{B} = 1/\beta.$$
 (.28

$$S = R = R / RP$$

$$RR^{B} = 1/\beta.$$
(.28)

$$S = R = R^B / RP \tag{27}$$

$$S = R = R^B / RP \tag{27}$$

$$R^{B} = \Pi / \beta$$
(.26)
$$S = R = R^{B} / RP$$

y-

, ,

$$\frac{I}{Y} = 1 - \frac{C^{E}}{Y} - \frac{C^{I}}{Y} - \frac{C^{P}}{Y} - \frac{G}{Y}$$
(.42)

C.4. Log-linearized model

C.4.1. Shadow rate representation

$$\widehat{y}_t = \frac{C^E}{Y}\widehat{c}_t^E + \frac{C^P}{Y}\widehat{c}_t^P + \frac{C^I}{Y}\widehat{c}_t^I + \frac{I}{Y}\widehat{i}_t + \frac{G}{Y}\widehat{g}_t$$
(.43)

$$\widehat{c}_t^p = \mathbb{E}_t \left(\widehat{c}_{t+1}^p - \widehat{s}_t + \widehat{\pi}_{t+1} - \widehat{\beta}_{t+1} \right)$$
(.44)

$$\widehat{i}_{t} - \widehat{k}_{t-1} = \gamma \left(\mathbb{E}_{t} \widehat{i}_{t+1} - \widehat{k}_{t} \right) + \frac{1 - \gamma \left(1 - \delta\right)}{\psi} \left[\mathbb{E}_{t} \left(\widehat{y}_{t+1} - \widehat{x}_{t+1} \right) - \widehat{k}_{t} \right] + \frac{1}{\psi} \left(\widehat{c}_{t}^{E} - \mathbb{E}_{t} \widehat{c}_{t+1}^{E} \right)$$

$$(.45)$$

1.

$$\begin{aligned} & / & \vdots \\ \widehat{q}_t &= \gamma^e \mathbb{E}_t \widehat{q}_{t+1} + (1 - \gamma^e) \Big(\mathbb{E}_t \widehat{y}_{t+1} - \mathbb{E}_t \widehat{x}_{t+1} - \widehat{h}_t^E \Big) + \Big(1 - M^E \beta \Big) \Big(\widehat{c}_t^E - \mathbb{E}_t \widehat{c}_{t+1}^E \Big) \\ & + M^E \beta \left(\mathbb{E}_t \widehat{\pi}_{t+1} - \widehat{s}_t \right) + (\beta - \gamma) M^E \widehat{\widetilde{m}}_t \end{aligned}$$

$$(.46)$$

$$\widehat{q}_{t} = \gamma^{h} \mathbb{E}_{t} \widehat{q}_{t+1} - \left(1 - \gamma^{h}\right) \widehat{h}_{t}^{l} + M^{l} \beta \left(\mathbb{E}_{t} \widehat{\pi}_{t+1} - \widehat{s}_{t}\right) + \left(1 - \frac{M^{l} \beta}{\mathcal{T}}\right) \widehat{c}_{t}^{l} - \beta^{l} \left(1 - M^{l}\right) \mathbb{E}_{t} \widehat{c}_{t+1}^{l} + (\beta - \beta^{l}) M^{l} \widehat{\widetilde{m}}_{t}$$

$$(.47)$$

$$\widehat{q}_t = \beta \mathbb{E}_t (\widehat{q}_{t+1} + \widehat{\beta}_{t+1}) + \left(\widehat{c}_t^P - \beta \mathbb{E}_t \widehat{c}_{t+1}^P\right) + (1 - \beta) \frac{H^E}{H^P} \widehat{h}_t^E - (1 - \beta) \frac{H^I}{H^P} \widehat{h}_t^I,$$
(.48)

:

$$\begin{aligned} \kappa &= (1-\theta)(1-\beta\theta)/\theta \\ 5. & / \\ \widehat{k}_t &= \delta \widehat{i}_t + (1-\delta) \widehat{k}_{t-1} \end{aligned}$$

$$\frac{B^E}{Y}\widehat{b}_t^E = \frac{C^E}{Y}$$

(.53)

Appendix D.

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