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reasonable parameters matching historical default experience.[2]

In this paper, we show that temporal variation in ambiguity (Knightian uncertainty)[3] carries important implications for the joint valuation of debt and equity; it creates an additional uncertainty channel that helps structural models match the historical equity premium and credit spreads while endogenously generating reasonable default rates. The role of time-varying ambiguity is fulfilled through its twofold effects on (uncertainty-adjusted) discount rates.

First, consider an exchange economy where a representative agent holds multiple priors (Gilboa and Schmeidler, 1989) about aggregate consumption dynamics. To make decisions robust to model misspecifications, the ambiguity-averse agent optimally evaluates investments according to the prior that leads to the lowest expected utility. This "worst-case" evaluation brings about a first-order effect of ambiguity on discount rates, wherein the magnitude is determined by the difference between the true expected growth rate and the worst-case belief used to price assets. Second, if the set of priors is updated over time, the uncertainty about future ambiguity levels also permits a second-order effect on discount rates, in the fundamentals. Second, it allows for time variations in both the market prices of uncertainty (the first-order effect) and the quantities of uncertainty (the second-order effect).[8] The resultant flexibility explains why our model outperforms existing ones in its time-series predictions.

2. Unlike these previous studies, which focus exclusively on investment-grade issuers as inspired by Huang and Huang (2012)'s findings, we examine the credit spreads of high-yield bonds as well. The latter is equally important in examining the credit spread controversy because hypotheticallhypothe

uncertainty about expected consumption growth[12]—which is absent in Drechsler (2013)'s model.

The remainder of our paper is organized as follows. In Section 2, we introduce the model and characterize the valuation of different assets in equilibrium. Section 3 describes how we measure economy-level ambiguity and illustrates the empirical relevance of the proposed ambiguity measure. Section 4 outlines the model estimation and discusses quantitative implications on asset pricing puzzles. Section 5 presents our conclusions.

2. Model framework

In this section, we build a general equilibrium model where the representative investor is uncertain about the correct probability laws governing the state process. We then allow Knightian uncertainty to vary in intensity over time and derive relevant testable implications on equity premium and credit spreads.

2.1.

Consider a measurable state space $(\Omega, \mathscr{F})$ where each $\mathscr{F} \in \mathscr{F}$ can be identified with a partition of Ω. Given a probability measure , $\mathscr{F}$ is the σ-field generated by a -dimensional Brownian motion defined on $(\Omega, \mathscr{F},)$. Suppose that the representative decision-maker does not know the true probability measure $_0$ and ranks uncertain consumption streams $= \{ \}^{\infty}_{=0}$, where $: \Omega \rightarrow \mathscr{R}$ is $\mathscr{F}$-measurable. To model preferences in the presence of uncertainty, we adopt the structure of recursive multiple priors:

$$^{*} = \min_{\in \mathscr{P}^{\Theta}} \left[\int^{\infty} (,) \mid \mathscr{F} \right], \tag{1}$$

where the set $\mathscr{P}$ of priors on $(\Omega, \mathscr{F})$ is constructed by means of Girsanov transformation. In particular, we can define the Radon–Nikodym derivative ( ) of with respect to the reference measure $_0$ through a density generator $(\vartheta\ )$:[13]

$$^{\vartheta} = - {}^{\vartheta} \vartheta \quad , \qquad {}^{\vartheta}_{0} = 1 \tag{2}$$

$$^{\vartheta} = \exp\left\{ -\frac{1}{2} \int_0 |\vartheta\ |^2 \quad - \int_0 \vartheta \right\}, \tag{3}$$

where is a Brownian

a large family of possible processes, whose limiting distributions are identical to that of an i.i.d. normal stochastic process with mean $\bar{\mu}$ and variance $\bar{\sigma}_{\mu}^{2}$, even after many observations of .

As discussed by Epstein and Schneider (2007), the multiplicity of probability measures in Eq. (1) indicates that the agent "has modest (or realistic) ambitions about what she can learn." In our model setup, it captures the ambiguous component $\mu_{\ ,\ }$, which is too poorly understood to be theorized about. In response to this model uncertainty, the agent forms a set of beliefs about expected consumption growth at time . We parameterize this set by an interval centered around the long-run mean $\bar{\mu}$,

$$\mu^{\mathscr{P}}_{\ ,\ } \in [\bar{\mu} - \ , \bar{\mu} + \ ], \qquad \geq 0, \tag{8}$$

because to the agent, the observed consumption growth is indistinguishable from a realization of geometric Brownian motion,

$$\frac{\ }{\ } = \bar{\mu} \ + \sigma \ _{\ ,\ }, \tag{9}$$

where $\sigma \ = \sqrt{\bar{\sigma}_{\mu}^{2} + \sigma^{2}}$. It follows that in Eq. (8) captures the level of ambiguity about the expected growth rates. That is, the higher is, the less confidence the agent has in her probability assessment of the growth rate and the larger the set of beliefs is. With the multiple-priors utility structure, time-invariant ambiguity ($\equiv$) points to the constancy of the priors' set, which in turn indicates the lack of learning from data. This constrained specification is termed "κ-ignorance" by Chen and Epstein (2002) and adopted by Miao (2009) and Jeong et al. (2015). Removing this constraint, this paper explicitly models the time variation in to allow the set of priors to actively respond to updates of information.

There are potentially many ways to obtain time-varying ambiguity endogenously through a detailed specification of the learning process, and Internet Appendix A provides a stylized model in this spirit. Specifically, this model features intangible information with ambiguous quality (Epstein and Schneider, 2008) and, more importantly, implies a stationary and persistent process of . The intuition is that the ambiguity-averse agent reacts asymmetrically to signals: she tends

Comparing the Hamilton–Jacobi–Bellman equation in Proposition 1 to that with traditional expected utility, we confirm that it is unnecessary to learn what the true μ , is to derive the model's implications. All that matters is the worst-case belief under which the agent's decisions are computed. Since Eq. (11) does not have a closed-form solution, we follow Benzoni et al. (2005) and CCG (2009) by approximating as an exponential affine function:

$$( ) \approx \;^{\eta_0+\eta_1} \;. \tag{13}$$

Appendix B shows that this approach provides an accurate approximation to the problem solution. It also shows that the price-consumption loading on ambiguity, η_1, is negative as long as the EIS is larger than one. This corresponds to the scenario wherein the agent recoups her investment in response to shocks that blur economic prospects. Consequently, asset prices tend to drop during times of high subjective uncertainty.

In equilibrium, state prices are shaped by the agent's marginal rate of substitution. In particular, given the maxmin representation (6), state prices are based on the worst-case density ϑ^*. The following propositionp

Finally, we assume a differential tax system: corporate earnings are taxed instantaneously at a constant rate of τ ; taxes has no loss-offset provision. Individual investors, however, pay taxes on interest income at rate τ and on dividend income at rate τ , but they are not taxed on their capital gains. It follows that the cash flows to the equity and debt issued at time $_0$, when the firm is solvent, are given by

$$\delta_{\ ,} = (1-\tau\ )(\delta\ -\ )-\quad +\ (\delta\ ,\ ), \qquad \text{and}$$
$$\delta_{\ ,} = {}^{-\ (\ -\ _0)}((1-\tau\ )\ +\quad),$$

respectively, where $\tau\ = 1-(1-\tau\ )(1-\tau\ )$ is the effective tax rate and the market price of newly issued debt.

Given that it is in the interest of the equityholders to choose when to default in such a way that the value of equity is maximized, the endogenous default can be formulated as an optimal stopping problem. Fix a domain $\mathcal{S} \subset \mathbb{R} \times (0,\infty)$ for the state vector $(\delta\ ,\ )$, and define $\tau_{\mathcal{S}} = \inf\{\ > 0; (\delta\ ,\ ) \notin \mathcal{S}\}$. Then the optimal default boundary is determined by finding a stopping time $\tau^*(\delta\ ,\ )$ such that

$$(\delta\ ,\ ) =$$

equity, which is the result researchers commonly pursue when attempting to resolve the equity premium puzzle. On the other hand, our model introduces an additional component in levered equity premium, as captured by the second term: the (present value) of tax benefits. Since tax benefits cannot be claimed after default, their value equals the product of the tax-sheltering value of coupon payments $((\tau \; - \tau \;) \;)$ and the probability of solvency $(1 - (\delta/\delta_0^*)^{\alpha} \;)$; the latter is directly determined by the default threshold $\delta_0^*(\;)$, whose relation with the ambiguity level is shown in Eq. (22). It follows that equityholders are more likely to lose the tax shelter when their marginal utility is high. Therefore, the model-implied levered equity premium is higher than its unlevered counterpart.

draw a comparison of $\tilde{}$ with alternative ambiguity measures.[26]

Fig. 1 shows the dynamics of $\tilde{}$, which is plotted as the dashed blue line.[27] We find that it widely fluctuates around an average value of 2.1%, ranging from 3.8% in the early 1990s recession to about 1.5% during most years under the Clinton administration. Moreover, taken together with

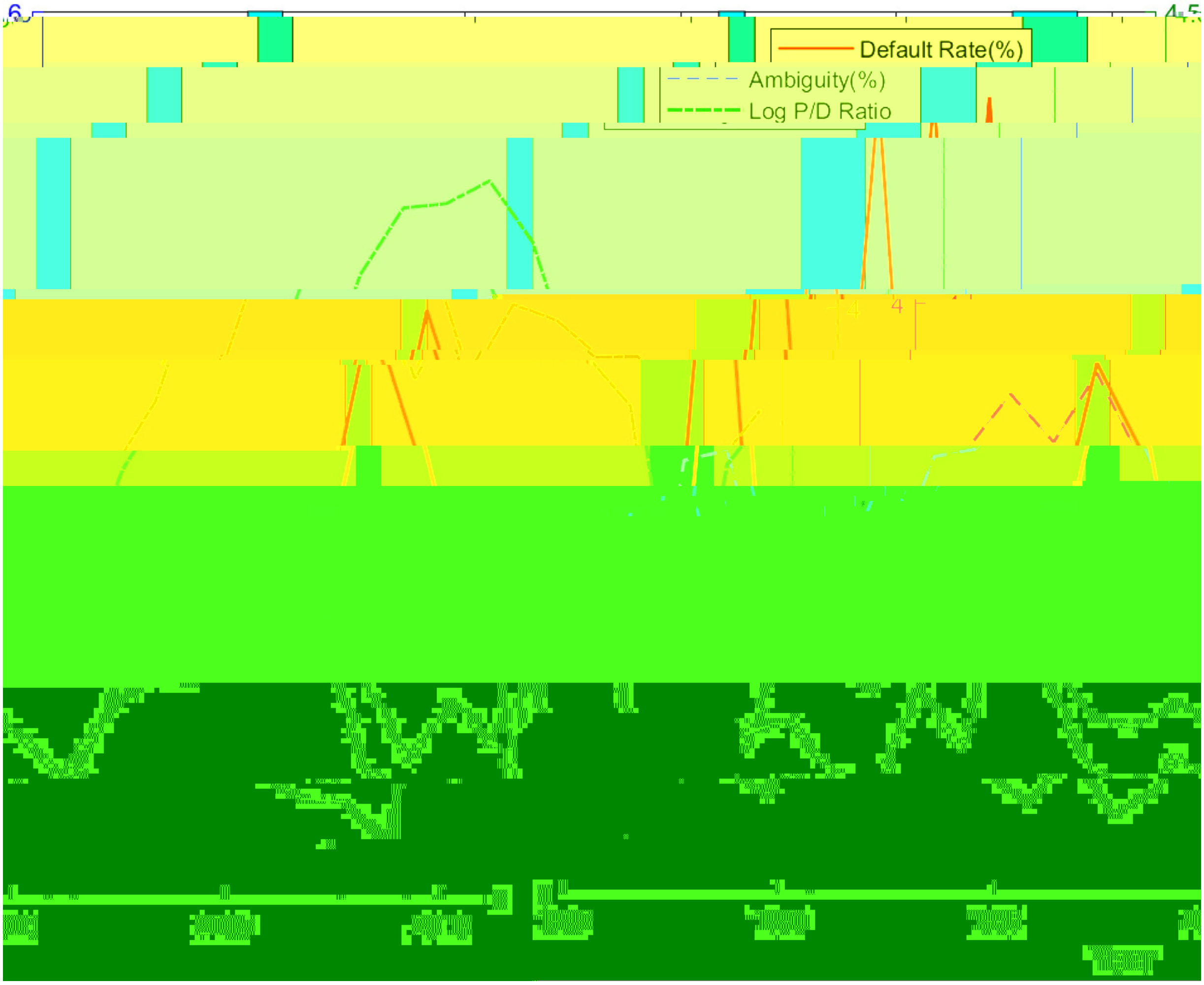

Fig. 1. Historical default rates, levels of ambiguity, and price-dividend ratios.
This figure shows the Moody's corporate default rates (solid red line), the annualized measure of ambiguity level constructed from professional forecasts (dashed blue line), and the logarithm price-dividend ratio on the NYSE/Amex/Nasdaq value-weighted index (dash-dot green line). The -axis on the left side applies to the first two time series, and the -axis on the right side the last. Shaded bars denote months designated as recessions by the National Bureau of Economic Research. The sample period spans from 1985 to 2010. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

output gap measured using the approach of Cooper and Priestley (2009), the ratio of new orders to shipments of durable goods (Jones and Tuzel, 2013), and the three-month growth rate of the Baltic Dry Index (Bakshi et al., 2012). We find that the economic and statistical significance of our ambiguity measure remains similar. Among the three conditioning predictors, the Baltic Dry Index accomplishes the greatest return predictability when combined with $\tilde{}$; the resultant adjusted ^{2} is 11.2%.

To further pin down the role of ambiguity in pricing corporate securities, we estimate the following predictive regressions of credit spreads for various rating classes:

$$_{+1} = \text{constant} + \tilde{} + \epsilon ,$$

where $_{+1}$ is the quarter-end yield spreads between Barclays (Lehman) US corporate bond and Treasury bond indices. Panel B of Table 1 contains the results of forecasting yield spreads of Aaa- to B-rated bonds. All regression coefficients are significant at the 5% level, and $\tilde{}$ seems to have stronger predictive power for speculative-grade bonds. The latter is not surprising, as ambiguity only affects the pure credit component in yield spreads, which accounts for a greater fraction in those lower rating categories (Huang and Huang, 2012; Avramov et al., 2007b; Rossi, 2014).

The predictive content of $\tilde{}$ is not limited to corporate securities. Panel C presents the performance of $\tilde{}$ in forecasting three-month excess returns on Treasury bond portfolios. We find that ambiguity explains a significant fraction of the variations in both intermediate-term and long-term bond returns. Again, the coefficient estimates are positive, indicating that in an ambiguous environment

Table 1

Asset return and credit spread predictability by the ambiguity level.

This table contains results from regressing (one-quarter-ahead) excess stock returns, credit spreads, and excess bond returns on the proposed ambiguity measure alone or with other predictors. Conditioning predictors include the output gap measure of Cooper and Priestley (2009), the new-orders-to-shipments ratio of Jones and Tuzel (2013), and the growth rate of the Baltic Dry Index (Bakshi et al., 2012). is defined as the difference between the continuously compounded return on the CRSP value-weighted index and the contemporaneous return on a three-month Treasury bill. - denote the credit spreads of Barclays (Lehman Brothers) bond indices. $_{+1}$ and $_{+1}$ are quarterly returns on the Barclays long-term and intermediate-term Treasury indices in excess of the contemporaneous return on a three-month Treasury bill. -statistics are computed following the procedures of Newey and West (1987) with a lag truncation parameter of six. The last two columns report Campbell and Thompson (2008)'s out-of-sample ^{2}s as well as -values of the Clark-West

When evaluating the unconditional predictive power of $\tilde{\ }$, we always compare it to the mean of all return/spread observations up to and including time- .

As shown in the seventh column of Table 1, out-of-sample ^{2}s are uniformly positive in univariate regressions. They suggest that using $\tilde{\ }$ as the predictor leads to a lower mean squared forecast error. Among different asset classes, the premiums on corporate bonds appear subject to the highest predictability, as in-sample results reveal. The -values from the Clark–West tests confirm the effectiveness of $\tilde{\ }$ as a real-time predictor.[32] The only exception is long-term Treasury bonds, for which the ambiguity measure is significant only if the test's size is set at 10%. Otherwise, none of the Clark–West -values exceeds 0.04. As to multivariate regressions, we carry out Clark–West tests [redacted] $\tilde{\ }$ lead to equal forecast accuracy.[33] The test results mirror those regarding univariate regressions: $\tilde{\ }$ contains signif- [redacted] significant at the 10% level. Generally, we find remarkable aspect of parameter estimates pertains to the persistence of the ambiguity level. The mean-reversion rate is estimated at 0.30, corresponding to a mean return time of more than three years; this finding substantiates our use of the perturbation approach for security pricing under slow-variation asymptotics of the ambiguity process. The unconditional expectation of ambiguity $\bar{}$ is about 1.11%. This quantity, when combined with estimates of σ , σ , and σ , implies that the proposed measure $\tilde{}$ has a long-run mean of 2.09%, which matches almost exactly the average value observed from the survey data. Finally, the estimate of σ is negative and significant at the 1% level.

The preference parameters reported in Table 2 are chosen to take into account the empirical evidence and economic considerations. The time discount factor β in the stochastic differential utility is usually calibrated at somewhere between 0.01 and 0.02.[36] Taking an intermediate value in this range, we fix β at 0.015. We further assume that the RRA coefficient γ is equal to 10, as do Bansal and Yaron (2004). The question of whether the magnitude of the EIS is greater than one is controversial. We set its value to Bansal and Shaliastovich (2013)'s estimate of 1.81, which they derive from bond market data as well as survey forecasts.[37]

.2.

In a step toward developing a unified understanding of how time-varying ambiguity affects the pricing of equities and corporate bonds, this section examines the model's implications for credit spreads. Using calibration methodologies commonly employed in the literature, we choose firm-level parameters to match the average solvency ratios of firms in different rating categories. However, in this calibration experiment, historical default rates are not targets toward which we calibrate the model. Instead, they serve as criteria used to assess the model performance. This procedure for generating endogenous default probabilities is consistent with BKS (2010).

.2.1. -

A key variable in the valuation of default claims is the cost of default ϕ. Given that the default timing and default boundary are endogenous in our model, the default loss, too, should be modeled to allow for covariation with the state of the economy. As will be illustrated in Section 4.2.3, our model indicates that the default timing tends to be positively correlated with the degree of ambiguity. It follows that financial distress would be enormously costly when prospects for economic growth are highly unclear (Shleifer and Vishny, 1992). Given this insight, we parameterize the default cost as a linear function of the ambiguity level and estimate relevant coeffi-

[36] It is calibrated at 0.01 by BKS (2010), at 0.015 by Chen (2010), at 0.0176 by Benzoni et al. (2011), and at 0.02 by Wachter (2013).

[37] Jeong et al. (2015), who address the issue of the sensitivity of this estimate to the inclusion of ambiguity aversion, estimate the EIS based on the utility structure that we employ in our study. Their estimate is generally higher than one as well, but it is accompanied by large standard errors.

Table 3
Calibration of corporate-level parameters.

Panel A reports the calibration values of parameters that do not vary among different credit ratings. τ denotes the corporate tax rate, τ the personal tax rate on dividends, and τ the tax rate on coupon income. Panel B shows some target parameters for individual firms' calibration by credit rating group. Debt-EBIT ratio is measured as (Short-term debt + Long-term debt) / EBITDA, corresponding to $/\delta$

[redacted]

Table 4

Model implicatio[redacted]

This table sho[redacted] by credit rating [redacted] Moody's Default & Recovery Database (DRD) from [redacted] over the 1985–2015 sample period. Column (3) shows the model-implied stationary default probabilities; the corresponding ten-year survival probabilities are obtained by solving backward Kolmogorov equations on a grid of ambiguity levels. The model-implied 90% confidence intervals displayed in Column (4) are derived from 5000 simulations for the realized cumulative ten-year default rates. Columns (5) reports median corporate yield spreads as listed in Table IA.E7. The corresponding spread values corrected for the non-default component are shown in Columns (6) and (7); they are firm-specific growth rates. As such, we can collect the realized ten-year default frequency for each cohort and then calculate the average rating-specific default rates in this hypothetical economy.[42] Column (4) shows the 90% confidence bands for the default rates derived from 5000 simulations. These confidence intervals encompass the historical default rates for all rating groups, suggesting that the modeled stationary default rates in Column (3) are reconcilable with the historical ones given the statistical uncertainty associated with ex post default rates.

While matching the historical default rates indicates how well the model quantifies the credit risk under the physical measure, the model-generated credit spreads assess the model's performance under the risk-neutral measure. One important empirical consideration is what kind of measure for history spread levels should be used for comparison. The standard practice in the literature is to set the firm's leverage equal to the average leverage of firms in a particular rating class and then compare model-implied spreads with the (weighted) average actual spread over a period. However, David (2008) and BKS (2010) are critical of this "averages-to-averages" comparison, as it is subject to convexity bias in credit spreads.[43] In this paper, we adopt a "medians-to-medians" strategy: first, the model is calibrated to the median financial ratios for a given rating group; second, the model's prediction is compared with the median spread across all bonds in that rating category. Internet Appendix shows that this approach is largely immune to convexity bias.

The model's implications for credit spreads are reported in the right panel of Table 4. It is well-known that noncredit factors, such as illiquidity in the bond market, account for a sizable portion of yield spreads. Thus, in testing the model's predictions, we correct the target spreads for the fractional liquidity components estimated by Longstaff et al. (2005, LMN hereinafter) and by Chen et al. (2018, CCHM hereinafter). For example, the total yield for a typical Baa bond is about 182 basis points, as reported in Column (5) (also see Table IA.E7 in the Internet Appendix); the estimate by LMN (2005) of Baa bonds' liquidity fraction is 29%. The credit component should account for $182 \times 0.71 \approx 129$ basis points, as shown in Column (6). As expected, the liquidity fraction of the total spread becomes smaller as the rating quality of the bond decreases, while

[42] Note that the default boundary is the same for these 765 firms because they start with the same initial debt-earnings ratio and are exposed to the same degree of ambiguity throughout the simulated sample. Feldhütter and Schaefer (2018) find that the downward bias in realized default rates is mainly attributable to the presence of systematic risk. Systematic risk is present in our simulated economy as well: first, the earnings process for individual firms contains a systematic component, as specified in Eq. (15); second, the variation in the (fractional) default boundary is driven by ambiguity shocks, which is common to all firms in the economy. Therefore, untabulated results indicate that our simulated ex post default rates share the same pattern as those of Feldhütter and Schaefer (2018); that is, their distribution is skewed to the right and their median is below the mean.

[43] An early version of Feldhütter and Schaefer (2018) raises the same point and attempted to quantify the convexity bias. Huang and Huang (2012) and CCG (2009) find that if structure models are calibrated to the historical default loss experience, the convexity effect is small and makes the historical spre0108 1a

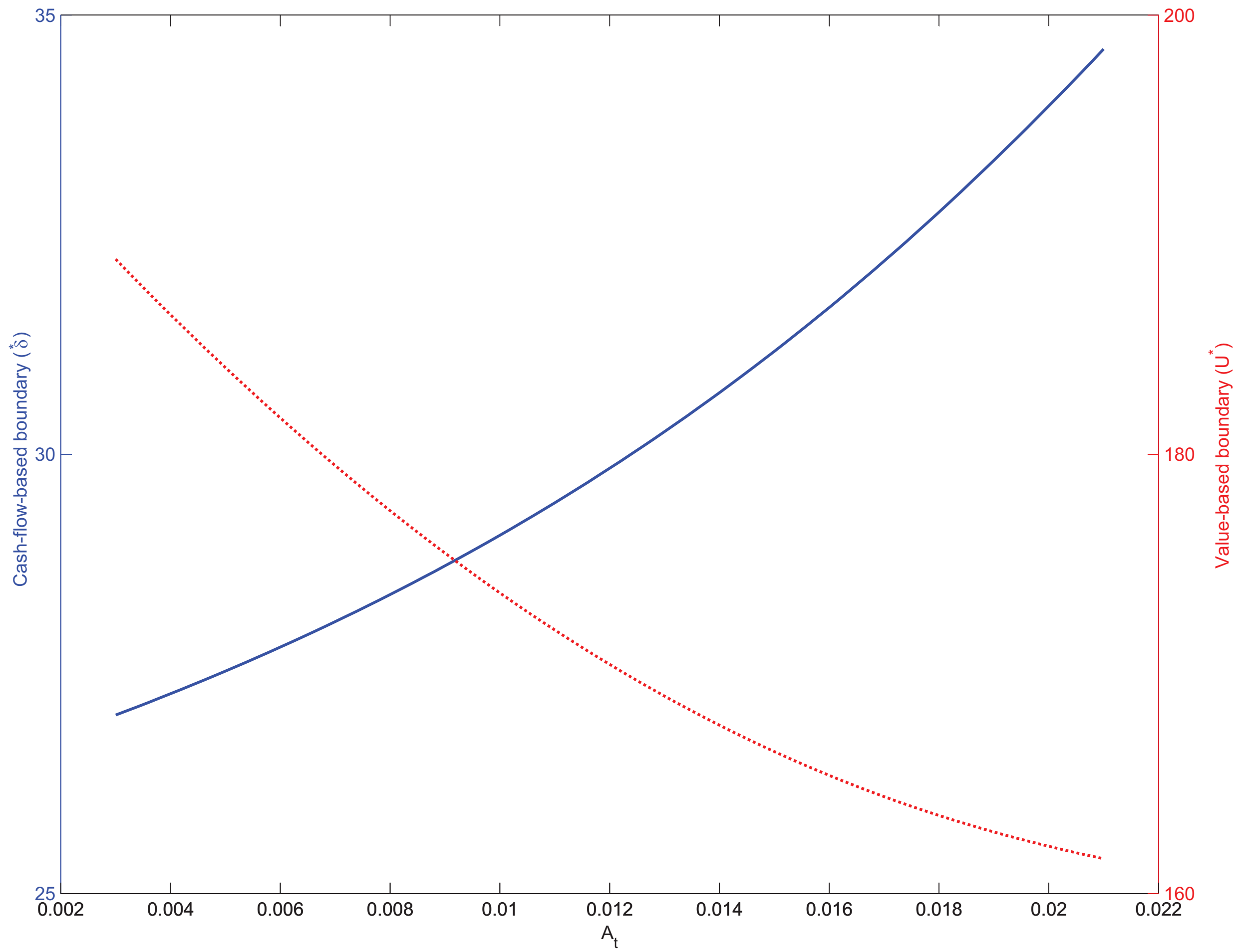

Fig. 2. Optimal default boundary as a function of the ambiguity level.
This figure plots the optimal default barriers for a typical Baa-rated issuer against the level of ambiguity. The solid blue line shows the default boundary in terms of the corporate earnings $\delta^*($ $)$. The dashed red line shows the unlevered asset value at default $U^*($ $)$. The optimal boundary is computed assuming the firm's cash flow starts at $\delta_0 = 100$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

his methodology, we test our model's implication by regressing sample estimates of boundary levels (based on firms that defaulted on their public bonds) on the ambiguity measure along with other theoretical determinants.[45]

The regression results are reported in Table 5, where the default boundary is expressed as a fraction of the face value of

Table 5
The effect of ambiguity on the default boundary.

This table reports cross-sectional regressions of the empirically observed boundary on the ambiguity level and other theoretical determinants. The default boundary is defined as the value of a firm's assets at default, scaled by the face value of debt. The regression coefficients are estimated with the Heckit model (Heckman, 1976; 1979). 1/ denotes the inverse Mills ratio obtained from the first-stage regression, which includes all independent variables at the second stage and the quick ratio. The values of -statistics are reported in parentheses. The sample period spans from 1983 to 2010.

~	−0.198	−0.146	−0.193	−0.175	−0.150	−0.192	−0.203	−0.194	−0.184
	(−3.450)	(−2.053)	(−2.796)	(−3.648)	(−2.606)	(−3.305)	(−3.551)	(−3.446)	(−2.998)
,		−0.091							
		(−6.723)							
			−0.452						
			(−0.211)						
,				−0.286					
				(−2.781)					
,					−2.845				
					(−3.232)				
,						0.322			
						(3.040)			
,							0.395		
							(0.553)		
								0.038	
								(0.837)	
,									0.245
									(1.088)
1/	−0.082	−0.105	−0.075	−0.047	−0.061	−0.119	−0.084	−0.086	−0.132
	(−0.793)	(−1.172)	(−0.682)	(−0.679)	(−0.727)	(−1.073)	(−	(−0.845)	(−1.562)
Const.	0.966	0.965	0.979	1.099	1.095	0.713	0.927	0.936	0.706
	(4.622)	(4.164)	(4.769)	(4.230)	(4.558)	(2.635)	(4.582)	(4.393)	(4.589)
Adj ^{2}	0.049	0.208	0.045	0.113	0.086	0.165	0.124	0.048	0.038
	230	230	230	230	225	114	227	230	144

Table 6
Simulated and sample moments of equity return and risk-free rate.

This table shows historical and model-implied unconditional moments of the equity return and the risk-free rate. In the "Data" section, Column (2) reports the return on the NYSE/Amex/Nasdaq value-weighted index and Column (3) contains the median descriptive statistics of monthly returns on all Baa-rated stocks listed on CRSP. The "Time-varying ambiguity" section reports the implications of our benchmark model, while the "Constant ambiguity" section shows a special case with ≡ ¯. Columns labeled "unlevered" and "levered" present the results on unlevered and levered perpetual claims to aggregate output. Column (8) corresponds to a typical Baa firm whose earnings growth rate is also subject to idiosyncratic shocks. The sample period spans from 1985 to 2010.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Data		Constant ambiguity		Time-varying ambiguity		
	Portfolio	Individual	Unlevered	Levered	Unlevered	Levered	Individual
(, −)	6.13	8.98	1.11	2.09	4.13	5.94	7.81
σ(,)	16.58	32.97	1.18	2.67	6.72	12.39	26.63
(, −)/σ(, −)	41.49	28.33	93.97	78.62	73.75	48.60	29.24
( )	1.21		1.70			1.18	
σ( )	1.18		0			0.92	

examining our model's ability to explain relevant empirical regularities. To this end, we focus on levered equity as a residual claim to earnings. As emphasized by BKS (2010), this approach introduces a default-risk dimension into the equity premium and preserves a direct link between asset pricing and firm-level decisions.

Column (2) of Table 6 reports real equity returns constructed using the CRSP value-weighted index. While the statistics are computed from the data for the 1985–2010 period, they appear fully consistent with the numbers for the entire postwar sample. Accordingly, model-implied moments are computed based on the unlevered and levered claims to aggregate output .[47] For levered equity, the initial ratio of debt to cash flow is calibrated at 2.61, the median value for all Baa firms. This is motivated by the finding of Avramov et al. (2007a) that the average rating for 3,578 public firms rated by S&P is approximately BBB.

Column (7) shows that the model-implied premium and volatility of levered equity are 5.9% and 12.4%, respectively, matching the data reasonably well.[48] Both are sub-

[47] To obtain the model's implications for the levered equity, we run 1000 simulations based on the Euler discretization. Each simulation generates 26 years' worth of daily observations. Simulating the model at lower frequencies makes the chance of getting negative values for higher than 0.001.

[48] While equity volatility is a bit below that found in the data, it falls within two standard errors of the empirically estimated volatility (the standard error of annualized volatility is estimated at 2.78%, based on the Newey–West adjustment with the lag length equal to six quarters). Also, because follows a CIR process, its conditional standard deviation rises

stantially greater than their counterparts for unlevered equity in Column (6) since the nature of levered equity as a residual claim increases the uncertainty in cash flows to shareholders. For comparison, we also present the implications of a κ-ignorance model. Columns (4) and (5) show that the restriction $\equiv \bar{}$ significantly lowers both the unlevered and levered equity premiums. To understand the contribution of time variations in , consider the expression for the former (which is in a closed form):

$$\frac{1}{} \left[\frac{U((1-\tau)\delta\,,\ )}{U((1-\tau)\delta\,,\ )}\right] - \ + \frac{(1-\tau)\delta}{U((1-\tau)\delta\,,\ )}$$
$$= \gamma\sigma\,\sigma\,\sigma \ + \frac{\sigma\,\sigma}{\sigma} \ + (1-\theta)\eta_1\xi_1\sigma^2 \ . \quad (24)$$

The first term in Eq. (24) reflects the standard risk exposure in a CRRA setting, and the second term represents the pure ambiguity premium, capturing the fact that Knightian uncertainty about the future payoffs is a first-order concern. These two terms are retained even in the κ-ignorance case. Numerically, however, they are subordinate to the third term, which is caused by the uncertainty associated with temporal variation in the ambiguity level, or more fundamentally, by the interaction of learning and ambiguity aversion. The learning model in Internet Appendix A sheds light on this result: the ambiguity level moves slowly with signals as the agent responds asymmetrically to good and bad news. The resultant permanence of the ambiguity process, as captured by the model parameter κ, raises the absolute values of η_1 and ξ_1 and thus amplifies the third term. Quantitatively, this is reflected in the 1.1% unlevered equity premium with time-invariant ambiguity, against the 4.1% with time-varying ambiguity. Also, the κ-ignorance model cannot generate excess equity volatility, as it posits that the volatility of unlevered equity equals that of dividends.

Our discussion so far leaves one important question unanswered: to what extent is the market index representative of a hypothetical Baa-level firm that is only exposed to systematic shocks? In response to this concern, we calibrate the model using the method described in Section 4.2.1: the firm's earnings follow Eq. (15) with the total volatility calibrated to 12.16%, which is the median volatility for Baa firms. These calibration results are then compared with the median values among all Baa firms.[49] As indicated by the results in Column (3), a typical Baa firm has both a higher mean excess return and a higher return volatility than the market index. This finding is in qualitative agreement with our model's implication shown in Column (8), as the inclusion of idiosyncratic cash flow risk increases the default risk embedded in levered equity.[50] Quantitatively, the model-implied equity premium and volatility for an individual Baa firm are largely consistent with their data counterparts. Moreover, the model reproduces the stylized fact that individual firm volatility is approximately twice the level of market volatility (CCG, 2009).

As demonstrated in CCG (2009), a key statistic to be matched in explaining the credit spread puzzle is the (rating-specific) average Sharpe ratio. In a departure from their procedures, where the firm-level Sharpe ratio is an explicit calibration target, we calibrate our model to corporate earnings volatility, which endogenously produces Sharpe ratios for individual firms. Based on equity returns, the median Sharpe ratio of Baa-rated firms is about 0.28, which is remarkably consistent with the 0.29 predicted by the model. If we consider only the systematic component in a Baa firm's cash flow, the model's prediction is roughly comparable to the Sharpe ratio for the market portfolio. However, when the ambiguity level is further restricted to a constant, the model-implied Sharpe ratio jumps to about 0.79. This reveals that the restricted model delivers unrealistically high returns (on levered equity) per unit of standard deviation.

The last two rows of Table 6 present the model's predictions about the risk-free rate. We find that the model-generated mean (1.18%) is almost identical to the historical counterpart (1.21%), but a degenerate set of priors implies a significantly higher level. As shown in Appendix B, the restriction $\equiv \bar{}$ lessens the impact of Knightian uncertainty, which therefore works only through misspecification doubts about expected consumption growth (or equivalently, only through the intertemporal substitution channel). By the same token, the κ-ignorance model implies no variation in the risk-free rate, whereas ours generates an annualized volatility of 0.92%. Hence, to match the empirical moments of the risk-free rate, there needs to be uncertainty about future levels of ambiguity, which contributes to the investors' precautionary savings motive.

. .

While several extant preference-based models capture levels of the equity premium and credit spreads (CCG, 2009; Chen, 2010; BKS, 2010), their ability to match the historical behavior of asset prices has not been fully demonstrated. An important advantage of our model lies in its ability to identify the time series of financial variables based on the empirical measure for ambiguity and the calibrated model. A comparison of these time series with their historical counterparts provides an intuitive way to evaluate the model's performance.[51] Fig. 3 presents the

with the square root of its level. Consequently, equity volatility is a nondecreasing and concave function of the ambiguity level. Given that the equity price is nonincreasing in , the model can reproduce "the leverage effect" (Black, 1976).

[49] The median is obtained from a July 1985 to December 2010 sample of BBB-rated stocks listed in the CRSP. The sample's beginning is determined by the first date for which credit ratings by S&P are available on the Compustat database. To be included in the sample, the stock must have at least 12 consecutive monthly return observations.

[50] Note that in the presence of idiosyncratic risk, the ambiguity level plays a limited role in determining the condition volatility of equity returns—a pattern that is consistent with empirical evidence that volatility asymmetry is generally stronger for aggregate market index returns than for individual stocks (Kim and Kon, 1994; Tauchen et al., 1996; Andersen et al., 2001).

[51] Appendix D considers other aspects of asset dynamics — the long-horizon predictability of equity returns and yield spreads as well as their correlations with consumption growth — and the performance of our models in capturing relevant empirical regularities.

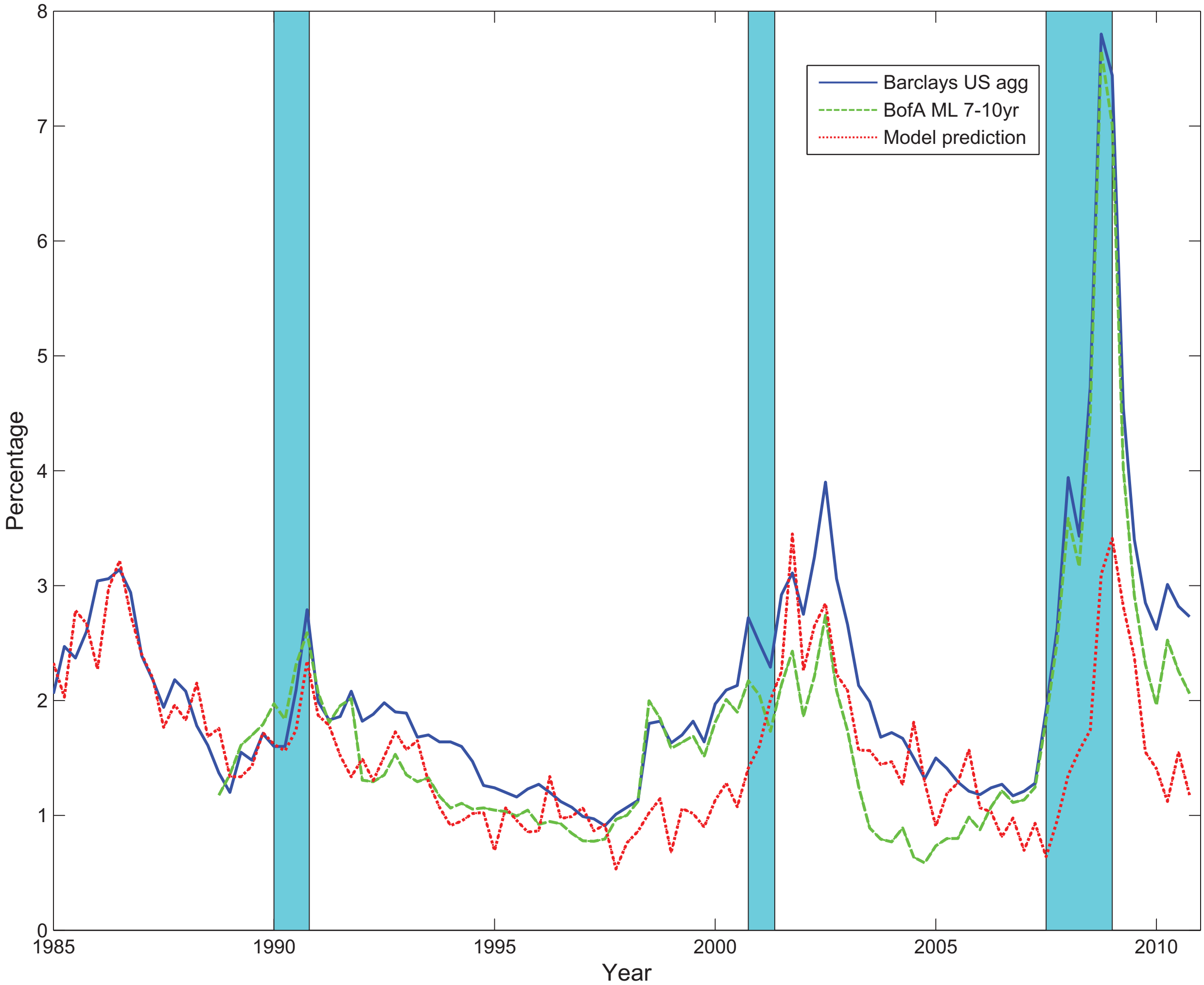

Fig. 4. Historical and modeled spreads in yields between baa and default-free bonds.
This figure displays the realized time variations in ten-year credit spreads for Baa bonds. The solid blue line shows the historical credit spread of Barclays US aggregate corporate Baa bond index, and the dashed green line corresponds to Bank of America Merrill Lynch US corporate BBB option-adjusted spread. The dotted red line shows the model-generated credit spread computed based on historical values of the ambiguity measure $\tilde{}$. Shaded bars denote months designated as recessions by the National Bureau of Economic Research. The sample period spans from 1985 to 2010. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the largest discrepancies between the observed and model should look beyond the business cycle when characterizing the dynamics of asset prices. In the framework of Chen (2010) and BKS (2010), the model's fit to the historical yield spreads is sacrificed through a discretization of expected consumption growth and consumption volatility. In this manner, they are able to deliver closed-form solutions to equity and debt prices with dynamic capital structure. Given this insight, an extension of our model to allow for endogenous capital structure presents an interesting topic for future research. Agency costs and transaction (debt retirement and reissuance) costs, as considered by Leland (1998) and Strebulaev (2007), can be incorporated into the extended model as well.

Another interesting direction for future research is to consider the implications of our model for derivative pricing. For instance, one can introduce secondary market search frictions (He and Milbradt, 2014) and evaluate the model's performance in fitting bond-CDS spreads (the non-default components of corporate yield spreads). Also, our model can be extended to shed light on stylized facts about option-implied volatility, that is, the "volatility smirk" and the variance premium, as studied by Drechsler (2013).[53]

Appendix A. More properties of the proposed ambiguity measure

This appendix supplements Section 3.1 with more details on

Table A1
A comparison of measures based on the dispersion of survey forecasts.

This table compares features of empirical measures that are constructed from the distributions of survey forecasts to proxy for ambiguity perceived by the representative agent or disagreement between heterogeneous agents. These measures include the ones examined in Anderson et al. (2009), Drechsler (2013), an early version of Ilut and Schneider (2014), Ilut and Schneider (2011), Buraschi and Jiltsov (2006), Buraschi et al. (2014), Yu (2011), and this article. The comparison table is divided into three panels. Panel A summarizes the formation of these measures and corresponding economic concepts in the model. The last row of this panel presents the -value each measure achieves when it is used to forecast one-quarter-ahead excess return on the CRSP value-weighted stock index. Panel B provides pairwise Pearson correlation coefficients for these empirical measures. Panel C focuses on the correlation between these measures and proxies for time-varying volatility.

	This article	Anderson et al. (2009)	Drechsler (2013)	Ilut and Schneider (2011)	Buraschi and Jiltsov (2006)	Buraschi et al. (2014)	Yu (2011)
Panel A: Summary of measures and their corresponding models							
Type of model	ambiguity aversion	ambiguity aversion	ambiguity aversion	ambiguity aversion	heterogeneous beliefs	heterogeneous beliefs	
The economic quantity about which the agent(s) feel uncertain / disagree	expected consumption growth	expected return on the market index	(a) drift of expected consumption growth; (b) expected consumption volatility; (c) intensity of jumps in expected consumption growth and in consumption volatility	expected innovation in TFP	expected consumption growth	expected consumption growth	
Statistics underlying the measure	interdecile range	weighted variance	standard deviation	interquartile range	standard deviation	mean absolute deviation	standard deviation
Data source	Blue Chip	SPF	SPF	SPF	SPF	I/B/E/S	I/B/E/S
-value from univariate predictive regressions	0.003	0.082	0.183	0.143	0.280	0.037	0.096
Panel B: Cross-sectional correlation among measures							
Anderson et al. (2009)	0.250						
Drechsler (2013)	0.479	0.192					
Ilut and Schneider (2011)	0.634	0.234	0.665				
Buraschi and Jiltsov (2006)	0.625	0.144	0.521	0.671			
Buraschi et al. (2014)	0.068	0.166	−0.004	0.032	0.216		
Yu (2011)	−0.038	0.024	−0.063	−0.046	−0.004	0.261	
Panel C: Cross-sectional correlation with volatility proxies							
,	−0.105	0.005	−0.057	−0.240	−0.274	−0.163	−0.147
,	−0.170	−0.217	−0.068	0.018	−0.198	−0.240	−0.085
,	0.302	0.053	0.296	0.354	0.459	0.205	−0.068
	0.391	0.021	0.350	0.401	0.413	0.050	−0.099
-value from predictive regressions when is controlled for	0.004	0.119	0.245	0.243	0.156	0.048	0.059

testing the implications of ambiguity for asset premiums, the choice about the modeling framework is less important than the robustness of the adopted empirical measure to outlying observations.

.2.

Outside of the literature of Knightian uncertainty, the dispersion of survey forecasts is also used to proxy for the level of disagreement in heterogeneous belief models. Interestingly, this stream of research contains at least two different approaches that imply opposite signs about the effect of belief heterogeneity on the equity premium. Specifically, the models of Miller (1977) and Chen et al. (2002) focus on the interaction between disagreements and short sales constraints and suggest that greater disagreement leads to lower subsequent returns. Empirically, Diether et al. (2002) find supportive evidence for this proposition by showing that earnings forecast dispersion negatively predicts future stock returns. The second approach, on the other hand, considers agents with identical preferences but different beliefs about economic prospects. In the absence of trading frictions, the equilibrium discount rate increases with the level of disagreement. This implied positive effect of forecast dispersion on the equity premium is supported by the time-series evidence in Anderson et al. (2005) and by the cross-sectional evidence in Buraschi et al. (2014, BTV hereinafter).

In this section, we attempt to distinguish our ambiguity variable ~ from measures for differences in beliefs. Since we do not explore in our model the implications of cross-sectional variation in the uncertainty about individual security, we focus our comparison on aggregate disagreement indices.[56] Regarding the disagreement-with-short-sales-constraint approach, we consider Yu (2011)'s measure, which can be regarded as an aggregation of Diether et al. (2002)'s or AGJ (2005)'s individual-level measures and a proxy for the disagreement about aggregate corporate earnings. Among studies with the second approach, we select the measures constructed by Buraschi and Jiltsov (2006) and BTV (2014). The former measure is the first principal component of differences in beliefs about six economic variables, where the difference in beliefs is defined as the cross-sectional standard deviation of SPF forecasts. The latter is similar to Yu (2011)'s measure in the sense that it is constructed bottom-up by aggregating disagreements regarding the earnings of individual firms. The biggest difference between them arises from the underlying statistics: Yu (2011) uses the standard deviation of analyst forecasts, whereas BTV (2014) adopt the mean absolute difference (the average of absolute differences between each pair of earnings forecasts).

Panel A of Table A.1 indicates that the measure of BTV (2014) is highly significant in forecasting one-quarter-ahead market returns, and Yu (2011)'s measure exhibits statistical significance at the 10% level. Unreported results show that both slope coefficients are negative, which is not surprising given that both measures are constructed bottom-up using the same dataset (I/B/E/S). While this is consistent with Yu (2011)'s interpretation, it is opposite to the implications from BTV (2014)'s model.[57] In contrast, the sign of the regression coefficient is positive for the Buraschi–Jiltsov measure, even though the null of no predictability cannot be rejected at any conventional significance level.

While both our ambiguity measure and aforementioned disagreement measures are extracted from forecast dispersion, they have distinct properties, the most noticeable being the underlying statistics and predictive ability. We find that the range-based measures seem to be exclusively used in studies with multiple-priors utility, for the reason explained in the last subsection. One may argue that standard-deviation-based measures for disagreements—which regarding risk premiums independent of stochastic volatility.

The results for other measures (based on forecast dispersion) as discussed above are also reported for completeness. We find that among these six measures, the one constructed by Buraschi and Jiltsov (2006) displays the highest correlation with time-varying volatility. For those with significant forecasting power for equity returns, their significance is generally unaffected by the inclusion of VIX. This finding suggests that volatility drives time variations in the asset premium through a risk channel that is distinct from those for time-varying ambiguity and heterogeneous beliefs.

Appendix B. Equilibrium prices and affine

Given the value function of the representative agent as in Eq. (6), to make (+ $_0$ (, (,)) under the most probability measure, we have

$$\mathfrak{D} \quad + \; (\; , \;) - \quad (\;) \qquad = 0.$$

Conjecture that has functional form in Eq. (12). Substituting it into the differential equation above, we ob-

$$1 - \;) \quad - \; - \frac{1}{2} \quad + \; — \quad - \frac{1}{2}$$

$$+ \frac{\mathfrak{D}}{\theta} + - - \qquad , \qquad , \quad / \; . \tag{B.1}$$

(11) in Proposition 1 is a special case with the correla- zero.

To solve the model, we employ Collin-Dufresne and (2005) log-linear approximation that essen- the expected squared approximation error. we seek an exponential affine expression for price-consumption ratio as presented in Eq. (13) such left-hand side of Eq. linear in except for $\theta/$ term. In case, parameters η_0 and η_1 can be by solving the problem:

$$\min \quad (\quad + \qquad - \qquad \eta_0 \;\; \eta_1 \quad) \quad ,$$

$$(1 - \quad) \qquad - \frac{1}{2} \qquad + \; (\qquad \bar{} \; - \quad),$$

$$(1 - \quad) \qquad\qquad - \frac{1}{2} \qquad\qquad\qquad - 1$$

$$- \qquad\qquad - \frac{1}{2} \qquad\qquad .$$

Fig. B.1 makes a comparison of this approximate price- ratio with one numerically solved from (11). These two solutions turn out to be fairly each other. when ambiguity level is around long-run mean, they almost indistinguishable other. This result demonstrates the accuracy of the affine approximation.

With first-order conditions for the optimal consumption under ambiguity, Chen Epstein (2002) drive the following SDF:

$$= \qquad (\quad) \qquad (\;\; , \;\;) \; ^{\theta} \; ,$$

where $^{\theta *}$ is the Radon–Nikodym corresponding to the worse-case prior. The preference is reflected in the presence of $^{\theta *}$: if $_0$ is the belief in the prior set $\mathscr{P}$, it degenerates one, and the becomes the standard one in Duffie and Epstein and Duffie and Skiadas (1994).

To prove Proposition 2, we apply the pricing kernel. It follows

$$= - \qquad —— \qquad = \qquad\qquad - \qquad \frac{1}{2}(1 + \quad)(1 + \qquad)$$

$$+ —(\;\; - 1) \qquad\qquad\qquad - \frac{\mathfrak{D} \quad ^{\theta}}{\theta} - \frac{\quad - 1}{} + \qquad .$$

Connecting it with Eq. (11), we the risk-free rate as a linear fu

$$=$$

where

$$= \quad / \quad + \quad - \frac{1}{2} \quad (1 + 1/ \tag{B.2a}$$

$$\varrho_1 = -1/\psi + \frac{1}{2}(\theta - 1)\eta^2\sigma^2$$

$$+ \Big((\theta - 1)\eta_1\sigma \; - \; ^{1}$$

derivative

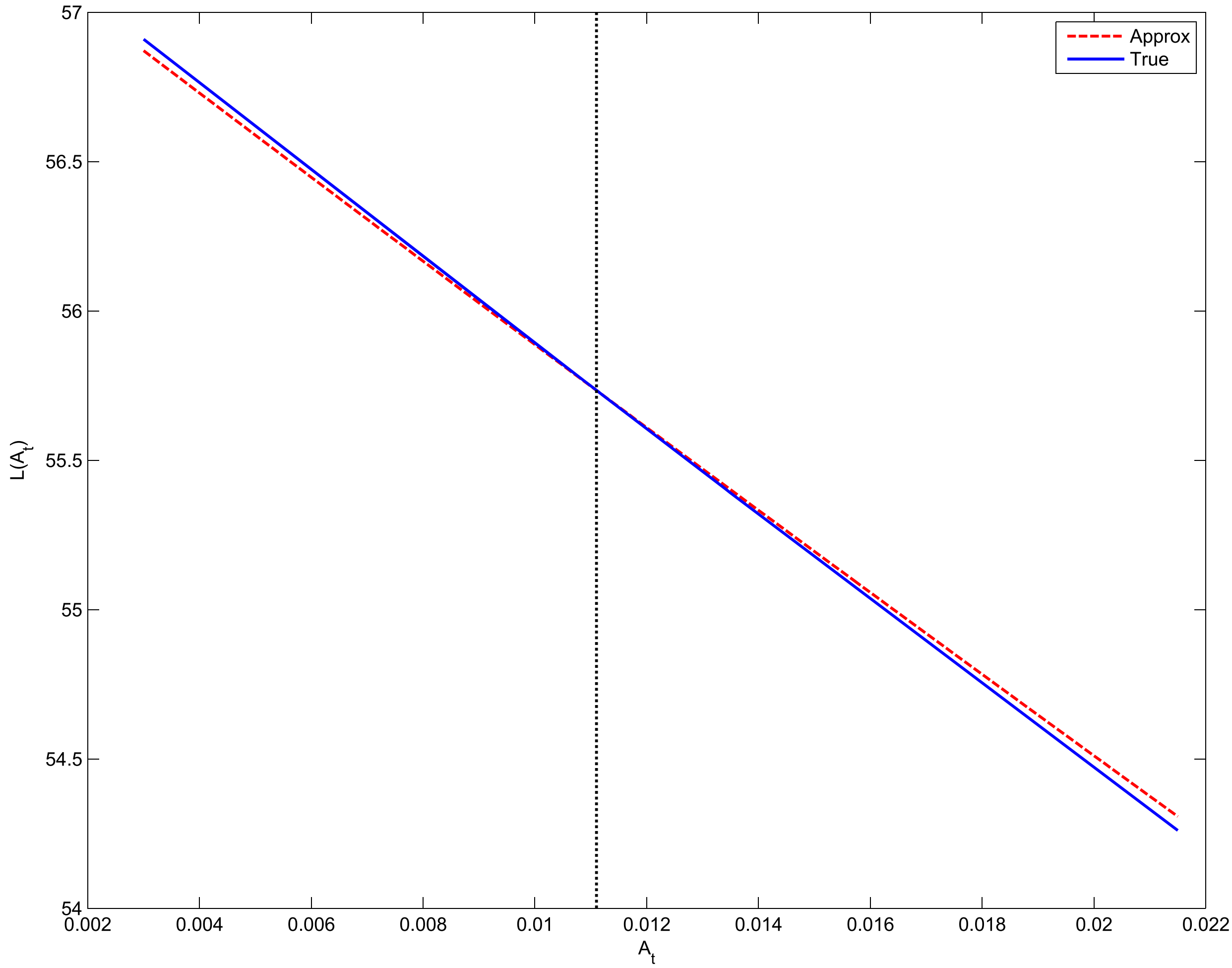

Fig. B1. Approximate solution for the price-consumption ratio.
This figure shows the accuracy of our approximation of the price-consumption ratio as an exponential affine function of the ambiguity level . The dashed red line displays the approximate solution for ( ) and the solid blue line the solution literally evaluated from the differential Eq. (11). The ODE is solved using the MATLAB function "ode45" with the initial condition $(0) = \frac{\theta}{(1-\gamma)(\mu - \frac{1}{2}\gamma\sigma^2)-\beta\theta}$. The grid line to the x-axis represents the historical mean of . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

time-varying ambiguity is crucial for the model to match the key statistics of risk-free rates.

Appendix C. A PDE-based characterization of equity and bond pricing

As a standard approach to endogenous default problems, we look for a model solution analogous to that in Leland (1994a,b), with the additional spatial variable . This type of free-boundary problem is usually solved by verifying that a given candidate function actually coincides with and a corresponding stopping time $\tau_{\mathcal{B}}$ is optimal. The following proposition is derived from the variational inequality verification theorem proved in Øksendal (2003).

3 . $: \bar{\mathcal{S}} \to \mathbb{R}$
$\in \mathcal{C}^1(\mathcal{S}) \bigcap \mathcal{C}(\bar{\mathcal{S}})$ ≥ 0 $\mathcal{S}$.

$\mathcal{B} = \{(\delta\ ,\ \ ) \in \mathcal{S};\ (\delta\ ,\ \ ) > 0\}$

.

(a) $\partial\mathcal{B}$.
(i) $(\delta\ ,\ \ )$ $\partial\mathcal{B}$, . .,
$\left[\int^{\tau_{\mathcal{S}}} \mathcal{X}_{\partial}\ (\delta\ ,\ \ )\ \right] = 0,$
(ii) $\in\ ^2(\mathcal{S}\backslash\partial\mathcal{B})$
$\partial\mathcal{B}$,
(iii) $\mathfrak{D}\ (\delta\ ,\ \ ) + \delta\ -\ (\ \ )\ (\delta\ ,\ \ ) \leq 0$ $\mathcal{S}\backslash\partial\mathcal{B}$,
$(\delta\ ,\ \ ) \geq\ (\delta\ ,\ \ )$ $(\delta\ ,\ \ ) \in \mathcal{S}$.
(b) ,
(iv) $\mathfrak{D}\ (\delta\ ,\ \ ) + \delta\ -\ (\ \ )\ (\delta\ ,\ \ ) = 0$ $\mathcal{B}$,
(v) $\tau_{\mathcal{B}} = \inf\{\ > 0; (\delta\ ,\ \ ) \notin \mathcal{D}\} < \infty$ $(\delta\ ,\ \ ) \in \mathcal{S}$,
(vi) $\{\ (\delta_\tau,\ \ _\tau); \tau \in \mathcal{T}, \tau \leq \tau_{\mathcal{B}}\}$ -
$(\delta\ ,\ \ ) \in \mathcal{S}$.
$(\delta_\tau,\ \ _\tau) =\ (\delta_\tau,\ \ _\tau)$ $\tau^* = \tau_{\mathcal{B}}$ -
.
(c) $\mathcal{U} = \{(\delta\ ,\ \ ) \in \mathcal{S}; \delta\ _,\ > 0\}$.
$(\delta\ ,\ \ ) \in \mathcal{U}$ $\mathcal{W}$ $(\delta\ ,\ \ )$
$\tau_{\mathcal{W}} = \inf\{\ > 0; (\delta\ ,\ \ ) \notin \mathcal{W}\} < \infty.$ $\mathcal{U} \subset$

$\{(\delta, \ ) \in \mathcal{S};\ (\delta, \ ) > 0\} = \mathcal{B}.$ -
$(\delta, \ ) \in \mathcal{U}.$

These results are deduced from the "high contact principle" (Øksendal, 1990; Brekke and Øksendal, 1991), which has been fruitfully applied in optimal stopping and stochastic waves. To find a reasonable guess for the continuation region $\mathcal{B}$, we refer to Proposition 3(c). In view of the expression for $\delta_{,}$, which is a nondecreasing function of δ, we conjecture that $\mathcal{B}$ has the form $\mathcal{B} = \{(\delta, \ ); \delta > \delta^*(\ )\}$ such that $\mathcal{U} \subseteq \mathcal{B}$.[61] In other words, inspired by Goldstein et al. (2001) and Strebulaev (2007), we define the default time in terms of corporate earnings. Therefore, the optimal strategy of shareholders is to find out the critical default barrier δ^* that depends on the current level of ambiguity.

Proposition 3 characterizes the maximized equity value by the PDE listed in ( ). The relevant boundary conditions derive from its $\mathcal{C}^1$-property. The continuity condition implies that

$$(\delta^*(\ ),\ ) = 0. \tag{C.1}$$

[redacted]

The upper boundary conditions (C.4) and (C.5) imply that when the firm's payoff grows to infinity, the possibility of default becomes meaningless so that the debt value tends toward the price of a default-free claim to the continuous cash flow $(1-\tau)\ \ +\ ^2$. The expression $U(\delta,\ )$ in Eq. (C.3) denotes the value of unlevered assets that can be written as

$$U(\delta,\ ) = \delta\ \ (\ ),$$

where ( ) is the price-earnings ratio. Given the affine model structure, both $\mathcal{PV}(\ )$ and ( ) would be solved with a log-linear approximation:

$$(\ ) \approx \ ^{\xi_0+\xi_1}\ , \tag{C.6}$$

$$\mathcal{PV}(\ ) \approx \ ^{\zeta_0+\zeta_1}\ . \tag{C.7}$$

Solutions to ξ_0 and ξ_1 can be derived in the same manner as in the case of the

[61] This conjecture is easily verified using the results established in Mordecki (2002) for a general Lévy process.

[62] While the continuous pasting condition (C.1) is necessary, the smooth pasting condition (C.2) need not hold in general, although it holds in the optimal default problem. See Alili and Kyprianou (2005) for a detailed discussion.

Table D2
Correlations of consumption growth with equity returns and credit spreads.

This table shows historical and model-implied correlations of consumption growth with stock returns, and with changes in credit spreads, for different leads and lags. The column labeled "Data" in Panel A reports the correlation between the growth rate of aggregate consumption and returns on the NYSE/AMEX/NASDAQ value-weighted index. The "Data"

Campbell and Cochrane (1999). To account for this weak correlation, exogenous shocks to the economy, besides consumption shocks, are necessary. With innovations in the ambiguity level, the model implies a mean correlation of 0.22 and a 90% confidence interval, which embrace the empirical correlation coefficients.

When studying consumption's correlation with credit spreads, we examine the spreads for both investment-grade and high-yield bonds. To disentangle variations in the credit components of yield spreads, Panel B focuses on the results of the lowest investment-grade rating, Baa, and the highest speculative-grade rating, Ba. In the data, there is a negative but weak correlation between spread changes and economic fundamentals. In our model, changes in credit spreads result only from uncertainty in ambiguity innovations, and as such they should be orthogonal to consumption growth without the correlation term. Incorporating the empirical comovement between the consumption and ambiguity processes leads to correlation coefficients of −0.20 and −0.25. The higher model-implied correlation for high-yield bonds (compared to investment-grade

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